# Plasticity and Deformation Process

Response of materials to stress

In the physically stressful environment there are three ways in which a material can respond to external forces:

- It can add the load directly onto the forces that hold the constituent atoms or molecules together (the main mechanism in crystalline ceramics)
- It can feed the energy into large changes in shape (the main mechanism in noncrystalline polymers)
- It can flow away from the force to deform either semipermanently (as with viscoelastic materials) or permanently (as with plastic materials)

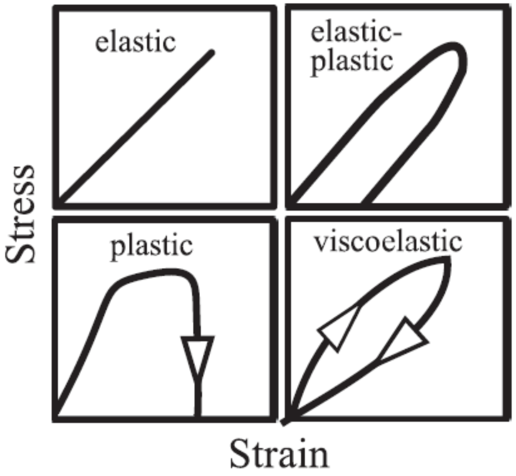


Figure 1.6. Stress–strain curves illustrating different types of behavior.

# Elastic (Hookean) Materials

In a material at equilibrium, in the unloaded state, the distance between adjacent atoms is 0.1 to 0.2 nm.

At this interatomic distance the forces of repulsion between two adjacent atoms balance the forces of attraction

When the material is stretched or compressed the atoms are forced out of their equilibrium positions and are either parted or brought together until the forces generated between them, either of attraction or repulsion, respectively, balance the external force

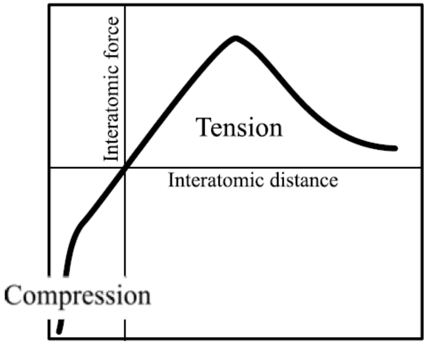


Figure 1.1. Stress–strain curve at the atomic level for a “perfect” material. The origin represents the equilibrium interatomic distance. On either side of the origin the curve is nearly straight.

With most stiff materials the extension or compression is limited by other factors to less than 10% of the bond length

When the load is removed, the interatomic forces restore the atoms to their original equilibrium positions

A crystal consists of a large number of atoms held together by bonds.

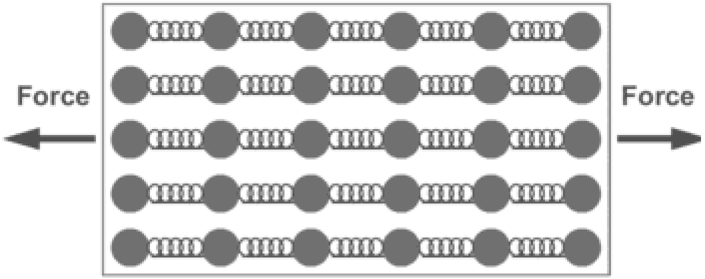
The behavior of the entire crystal in response to the force is the summed responses of the individual bonds.

Hooke described this phenomenon as “ut tensio, sic vis” or “as the extension, so the force.”

In other words extension and force are directly and simply proportional to each other, and this relationship is a direct outcome of the behavior of the interatomic bond

When we measure the response of the crystal to a stress, what is actually measured is the increase in length of the whole sample. We assume that the material is homogeneous and one part will deform as much as the whole to obtain the general relation between the stress and the strain of the material:

$$\sigma = E\varepsilon \qquad \text{Hooke's law}$$



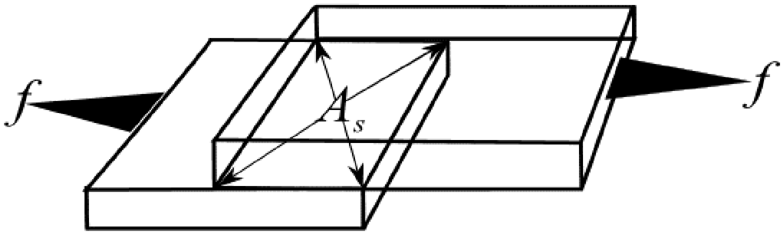
The characteristic of Hookean materials is that they are elastic.

They can be deformed within the elastic limit and will return to their original shape almost immediately after the force is removed (almost immediately because the stress wave travels through the material at the speed of sound in that material)

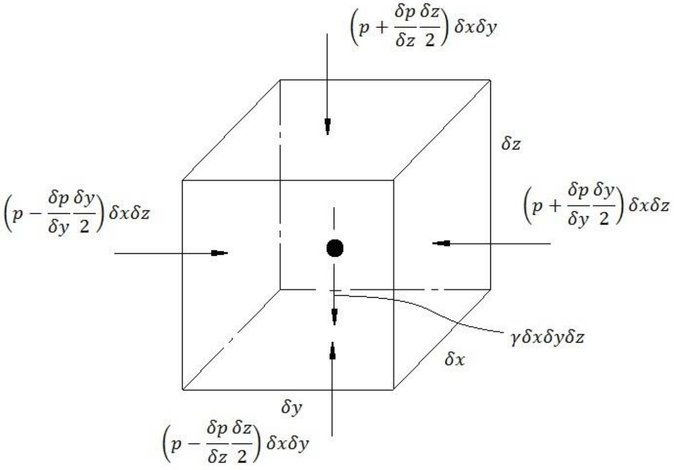
Young’s modulus is a measure of stiffness in simple extension or compression

There are other types of deformation that have different effects on the interatomic forces and therefore different effects on the material

Shear stress $\tau = G\gamma$



Hydrostatic stress $\sigma_h = -ke$



# The Energy of deformation

When material is deformed , strain energy is stored in the deformation of its bonds
This energy brings the material back to (or close to) its original shape (energy can be dissipated in a number of ways, such as heat, sound, surface energy, plastic deformation, or kinetic energy)

Stains in a Hookean material are relatively small, and all the energy is stored in stretching the interatomic bonds, termed the internal energy, U.

If the material is made of relatively long and unrestrained molecules, the energy can also be stored in changes in their shape and mobility, termed the entropic energy, S.

$$A = U - TS$$

where A is the Helmholtz free energy, U is the internal energy component, and –TS is the entropic component, made up of temperature, T, and entropy, S.

If you increase the temperature of a material that relies on internal energy for its elastic behavior (like ceramic), it will expand

An entropic energy based material (like rubber) will contract upon increase of temperature.

Elastomer response

A technical rubber is composed of very long chains (molecular weight of about $10^5$) of one or more monomer units, with each unit more or less freely joint to the chain, so that each joint allows a wide range of movement.

This free rotation about the bonds of the backbone is what distinguishes a rubbery polymer from a crystalline one: in a crystalline polymer (or in areas of crystallinity) the units cannot move freely because they are packed so closely and rubbery behavior is impossible.

As the temperature increases, the movement of the molecules and their subunits becomes more and more rapid. Conversely, as the temperature decreases, the activity of the molecules slows until, finally, at a temperature dependent on the type of rubber, it ceases altogether and any force that is exerted on the rubber meets the resistance of the covalent bonds linking the atoms.

A rubber at the temperature of liquid nitrogen is Hookean and is glassy. The temperature at which this phenomenon occurs is called the glass transition temperature.

At normal temperatures the rubber chains rotate in Brownian motion. It is this rotation that produces the tension and elasticity of rubber. When we hold a rubber and stretch, we do work on the rotating molecules to decrease their entropy. Rubber resists stretching more at high temperatures.

# Viscolelasticity

All materials are viscolelastic to some extent
They give both viscous (liquid-like) and elastic (solid-like) responses to stresses

Hence all materials flow

‘παντα ροι’ (panta roi) or ‘everything flows’ as Heraclitus said

‘the mountains flow before the Lord’ as prophetess Deborah said

Mountains flow if the time scale of observation is extended to geological scale.

Using an every-day time-scale we can define a solid as a material which either does not flow or flows plastically.

Plastic flow requires a stress larger than a definite stress, the yield stress, and a plastic solid does not flow under the action of a stress below that yield stress.

If we observe the objects for a very long time we see that the limiting stress below which the materials do not flow is quite low

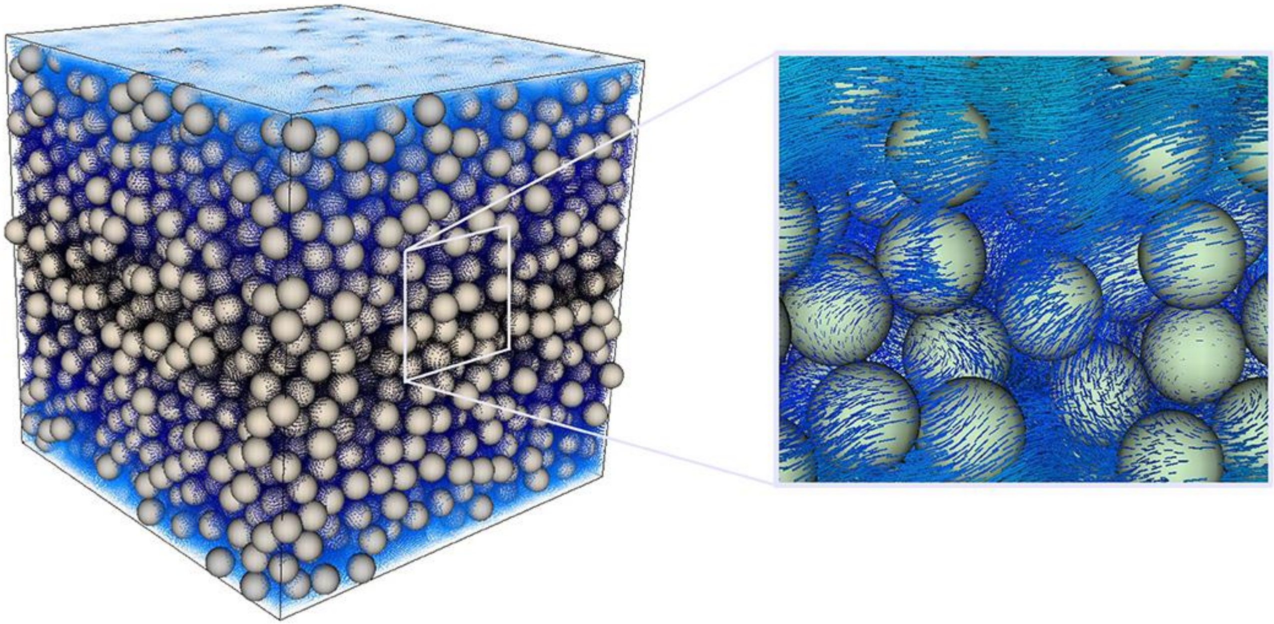
The difference between a solid and a liquid material is only temperature

The response of matter under stress at high temperature for a short time is identical to the response of matter under stress at low temperature for a long period of time

Hence solids need more time to respond to stresses and to flow

Let’s consider homogeneous suspensions that consist of both liquid and solid as model to analyze the flow behavior

Suspensions exhibit different degrees of elasticity and viscosity depending on their structure

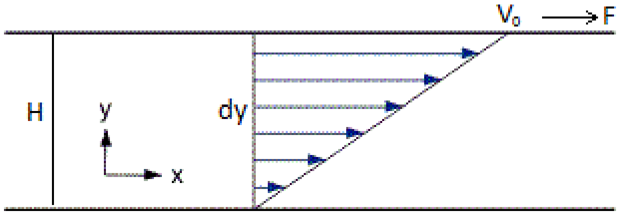


Consider water as the binder phase of the suspension. Its resistance to flow under stress is defined as its viscosity, η.

$$\tau = \eta\dot{\gamma}$$

where $\tau$ is the shear stress (Pa) and $\dot{\gamma}$ is the shear rate (1/s)

Simple shear flow of water between two plates of infinite lengths deforms the liquid according to Newtonian flow:



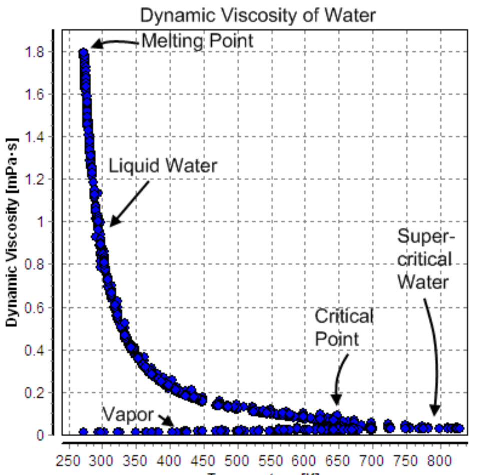
Velocity gradient is linear $V_z = \frac{V_0}{H}$

Shear stress $\tau_{yz} = \frac{F}{Area}$

Shear strain rate $\dot{\gamma} = \frac{\partial V_z}{\partial y} = \frac{V_0}{H}$

$$\eta = \frac{\tau_{yz}}{\dot{\gamma}} = \frac{F * H}{A * V_0}$$

Experimental Data Points from Dortmund Data Bank



Water behaves purely viscously because it does not contain solid and structure

Consider alumina powder as the filler of the suspension

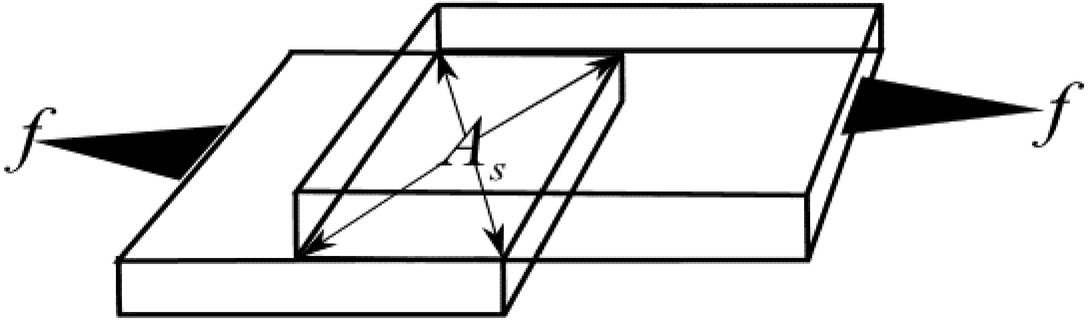
Alumina is a hard, brittle ceramic and behaves mostly like an elastic solid under shear stress:

$$\tau = G\gamma$$

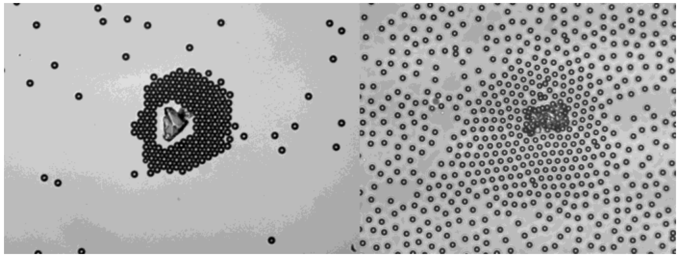
where $\tau$ is the shear stress, G is the shear modulus, $\gamma$ is the shear strain

$$\tau = \frac{F}{Area}$$

$$\gamma = \frac{dx}{dy}$$



The suspension is a homogeneous mixture of water and alumina, powders are well dispersed in water without agglomerations



By virtue of the interactions between the binder and filler particles (surface adhesion, wetting, hydration), the flow behavior of the mixture changes to non-Newtonian viscosity:

$$\tau = \mu \dot{\gamma}^n$$

where $\mu$ is the non-Newtonian viscosity, n is the consistency index. Shear thinning occurs when n<1.

Non-Newtonian fluids respond with a more or less than doubled shear rate when the force is doubled, depending on whether they show shear thinning or shear thickening.

Most aqueous ceramic suspensions show shear thinning, so that doubling the force will more than double the shear rate, thus making deformation of the material relatively easier at higher shear rates.

$$\mu = \frac{\tau_{yz}}{\dot{\gamma}^n} = \frac{F * H^n}{A * {V_0}^n}$$

The stress-strain relation of non-Newtonian suspensions has been analyzed using Maxwell's model:

A unit suspension is considered according to the model as the combination of purely viscous and purely elastic parts.

The shear stress of purely viscous damper is proportional to the shear rate

$$\tau_{yx} = \eta\dot{\gamma}$$

The shear stress of purely elastic spring is proportional to shear strain

$$\tau_{yx} = G\gamma$$

The relation according to the model is a combination of those:

$$\tau_{yx} + \left(\frac{\eta}{G}\right)\frac{d\tau_{yx}}{dt} = \eta\,\dot{\gamma}$$

Under steady flow, stress response of the suspension should be constant (independent of time) and the flow will be of viscous character

Under sudden flow, stress response of the suspension should be time dependent and $\frac{d\tau_{yx}}{dt} \gg \tau_{yx}$
So the suspension behaves like a pure elastic solid under sudden deformation.

$$\tau_{yx} + \left(\frac{\eta}{G}\right)\frac{d\tau_{yx}}{dt} = \eta\,\dot{\gamma}$$

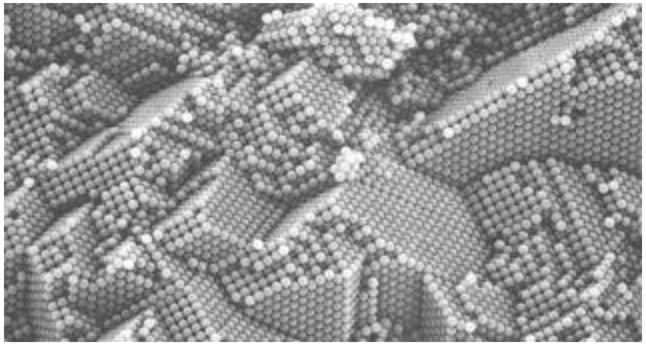
Maxwell model is applicable to all materials since all materials are viscoelastic to some extent

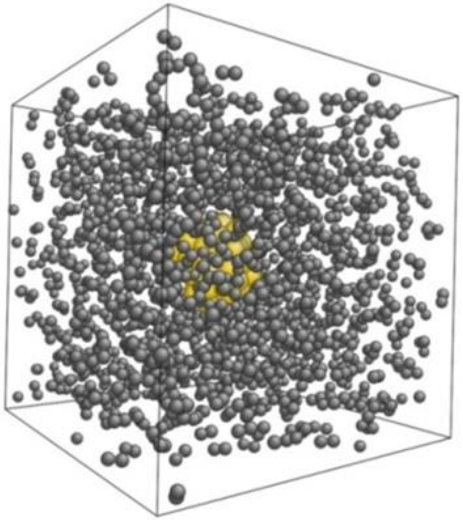
The ratio of the viscosity of a material to its shear modulus $\left(\frac{\eta}{G}\right)$ is termed the relaxation time λ

The relaxation time is basically the time it takes for the molecules in the unit cell of the material to liberalize from the effect of the surrounding stresses and return to equilibrium structure by Brownian motion.

λ determines the extent of elastic response of a material to stresses.

Low relaxation time means the material quickly returns to equilibrium structure after the stress is removed





Prophetess Deborah: ‘even the mountains flow before the Lord’, M. Reiner’s translation of Bible, Judges 5:5

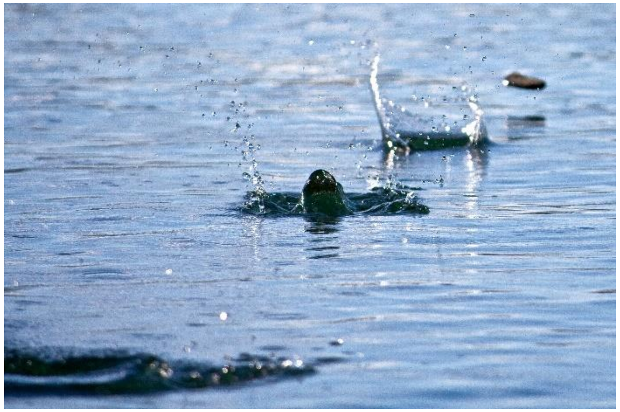
Deborah number is the ratio of the relaxation time to deformation time which is the inverse of shear rate:

$$De = \frac{\lambda}{t_d}, \qquad t_d \approx \frac{1}{\dot{\gamma}}$$

The higher the Deborah number (deformation time smaller than the relaxation time), the more elastic is the response of the material.

Molecular structure of a material is close to equilibrium conditions when the deformation or the deformation rate (inversely proportional to deformation time) is small

The response of the material to deformation in that region reflects the molecular arrangements



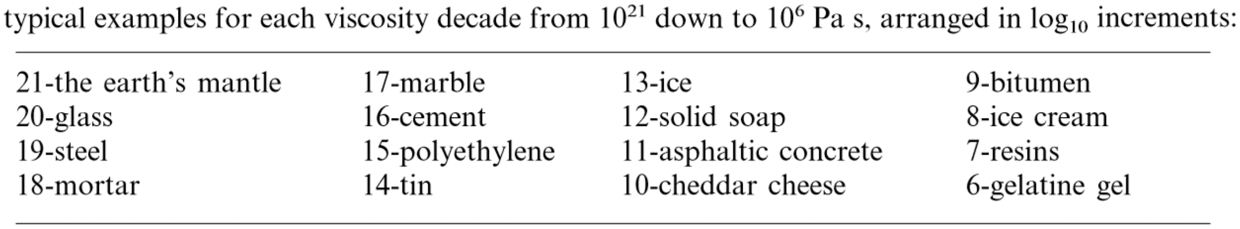
Nothing seems more solid-like than the rocks of the earth, but even they flow by creep

Geologists found the viscosities of many rock samples by observation for a long time

These very long time scales appropriate to geological studies mean that even very high viscosities and the resultant very low shear rates are significant.

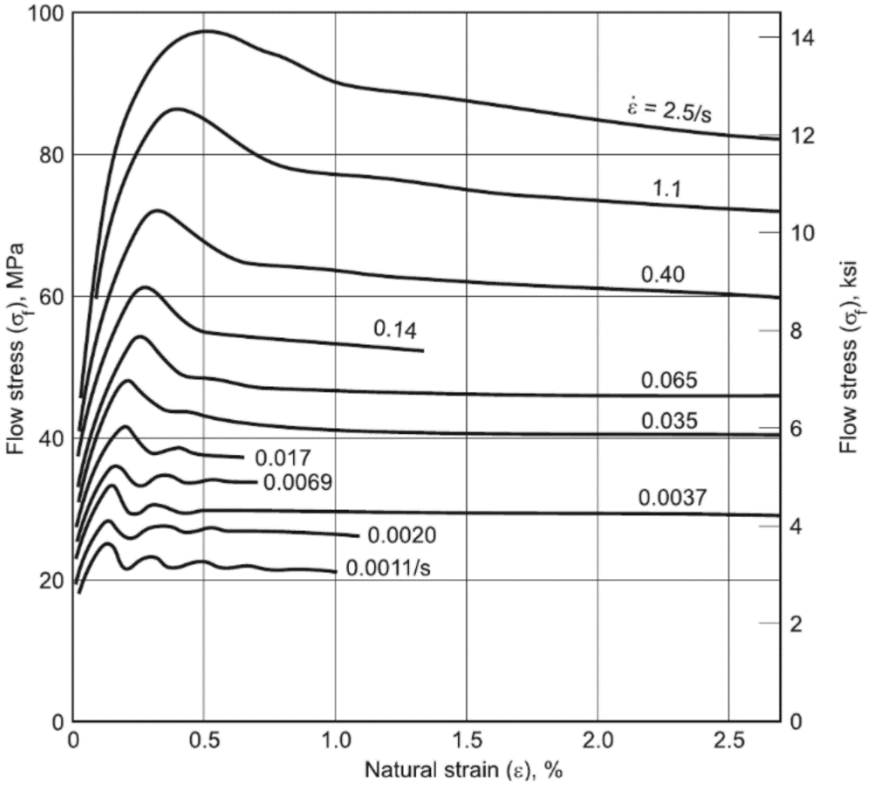
Long-term creep experiments on natural quartzites led to viscosities in the region of 3–4×$10^{15}$ Pa s

Cooled lava viscosity is $10^{12}$ Pa.s and the viscosities for lower crust and upper mantle of Earth are found as 3×$10^{19}$ and 3×$10^{18}$ Pa.s respectively



typical examples for each viscosity decade from $10^{21}$ down to $10^{6}$ Pa s, arranged in $\log_{10}$ increments:

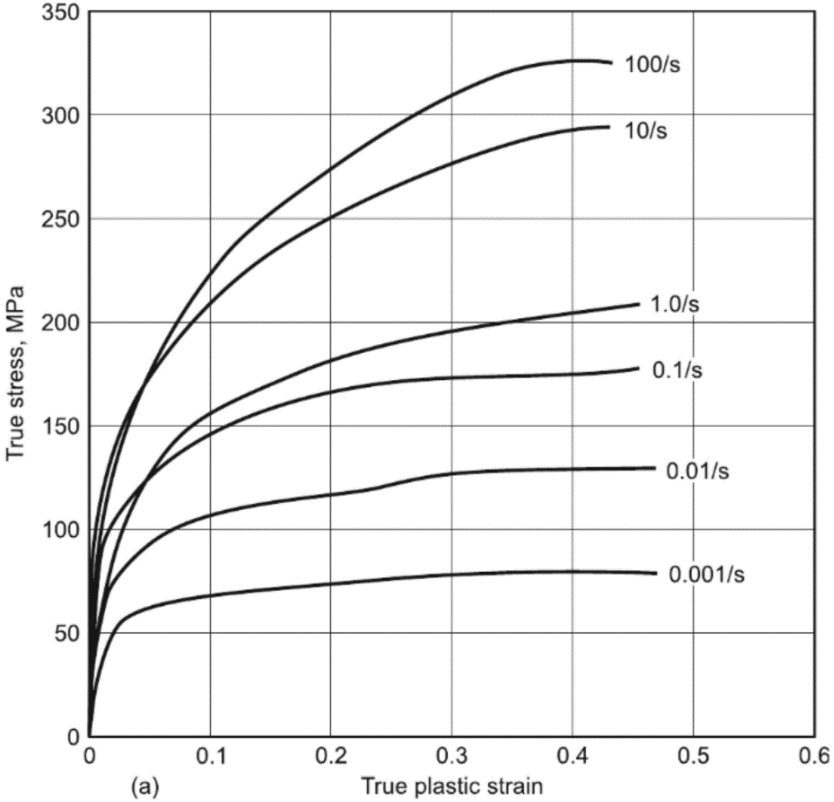
21-the earth's mantle
20-glass
19-steel
18-mortar
17-marble
16-cement
15-polyethylene
14-tin
13-ice
12-solid soap
11-asphaltic concrete
10-cheddar cheese
9-bitumen
8-ice cream
7-resins
6-gelatine gel



## CS.032 1025 carbon (0.25% C) steel, flow stress-strain curves at various strain rates

Temperature ($T$) = 1100 °C (2012 °F). Stress-strain curves show that at higher strains the flow stress is approximately constant. This is increasingly true at smaller strain rates ($\dot{\varepsilon}$). Curves were obtained in hot torsion experiments. UNS G10250

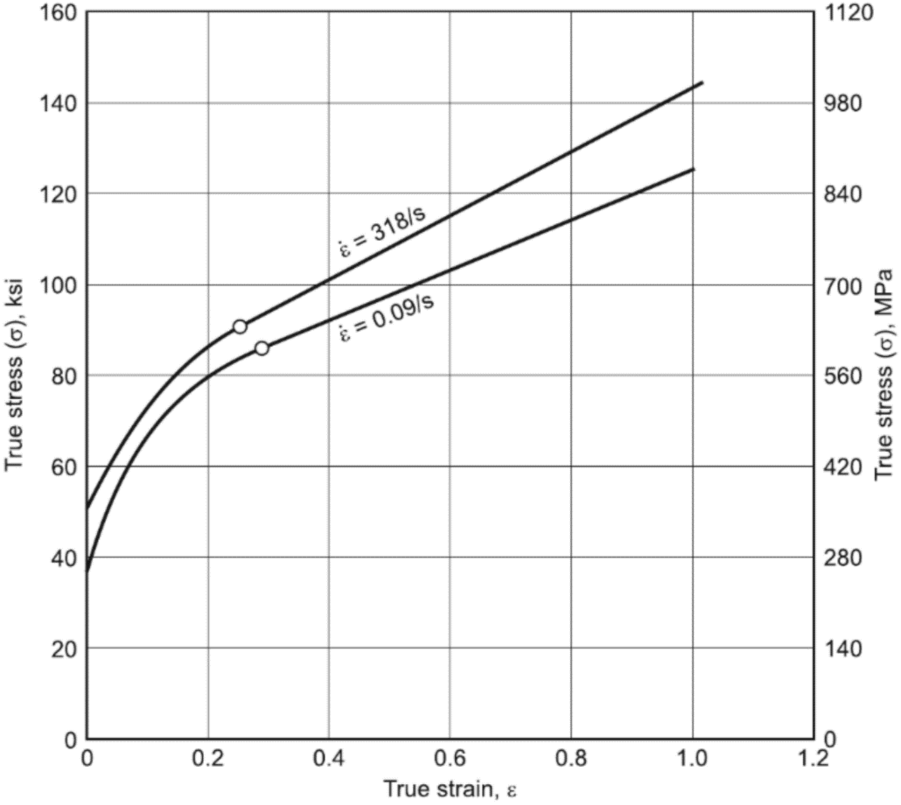
Source: K. Lange, Ed., *Handbook of Metal Forming,* McGraw-Hill, 1985, p 16.11



## HS.005 Microalloyed high-strength low-alloy (HSLA) steel, compressive true stress-true plastic strain curves at different strain rates

Hot rolled. Thermomechanical processing typically includes rough rolling, 1100–1240 °C (2012–2264 °F), and finish rolling, 810–900 °C (1490–1652 °F), fast cooling to 700 °C (1292 °F), and air cooling. (a) Tested at 900 °C. (b) At 1200 °C. Composition: Fe-0.08C-1.3Mn-0.3Si-0.2Ni-0.08V-0.05Nb-0.015P-0.008S

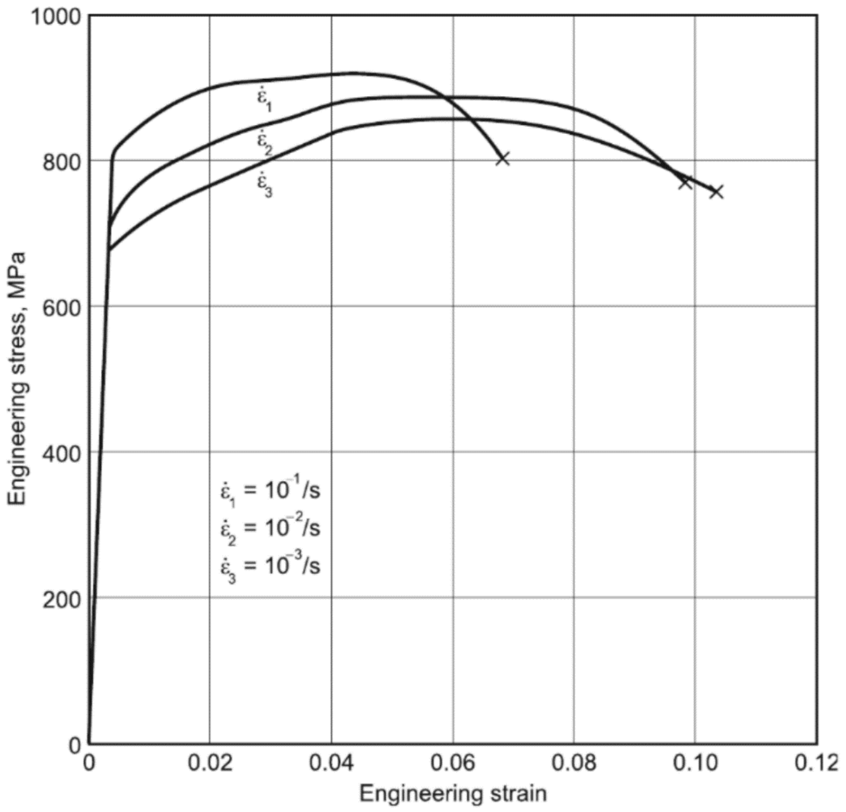
Source: N.S. Mishra, in *Hot Working Guide A Compendium of Processing Maps,* Y.V.R.K Prasad and S. Sasidhara, Ed., ASM International, 1997, p 337



## CS.038 1112 carbon steel, true stress-strain curves with effect of strain rate

True stress-strain curves for 1112 steel at different strain rates at 21 °C (70 °F). When metals are tested in tension at different strain rates, the flow stress corresponding to a given strain is found to increase with strain rate. The following equation is frequently used to relate flow stress and strain rate at a given strain and temperature: $\sigma = \sigma_1 \dot{\varepsilon}^m$, where $\dot{\varepsilon} = d\varepsilon/dt$ and $\sigma_1$ and $m$ are material constants. The exponent $m$ (strain-rate sensitivity) is found to increase with temperature, especially above the strain recrystallization temperature. In the hot-working region, metals tend to approach the behavior of a Newtonian liquid for which $m = 1$.

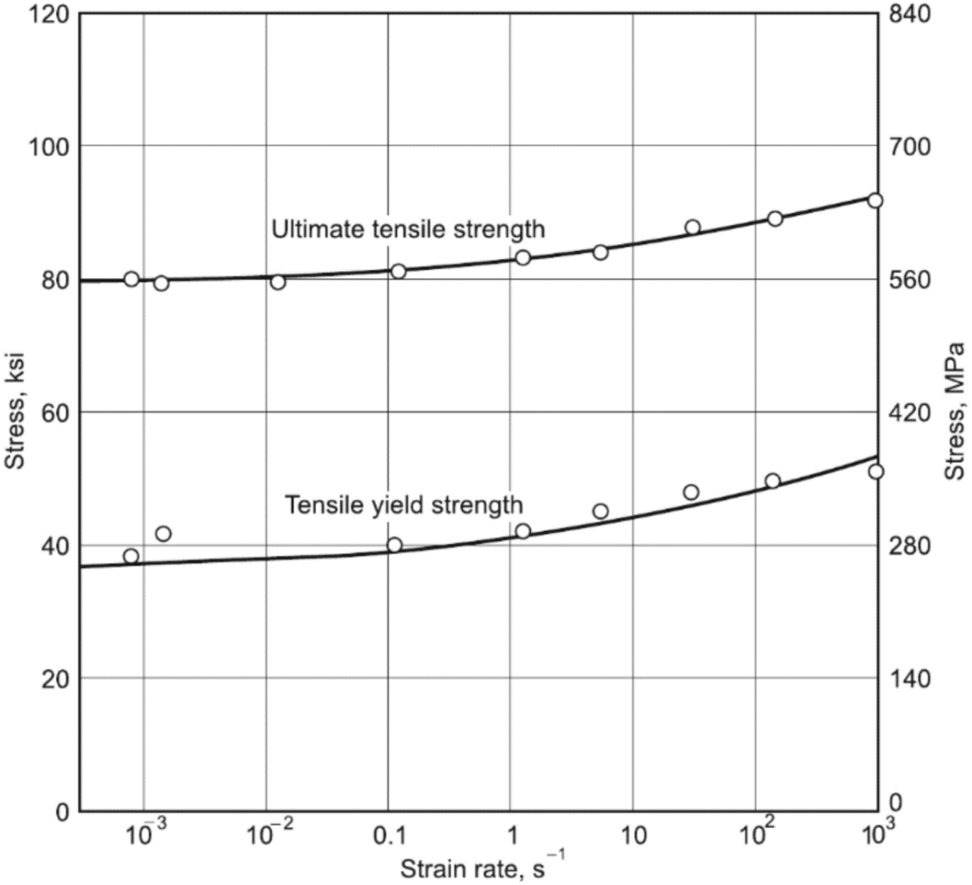
Source: M.C. Shaw, *Metal Cutting Principles,* Clarendon Press, Oxford, 1984, p 69



## CS.033 1040 carbon steel, engineering stress-strain curves with effect of strain rate

Effect of different strain rates on the tensile response. The yield stress and flow stresses at different values of strain increase with strain rate. The work-hardening rate ($m$), on the other hand, is not as sensitive to strain rate. This illustrates the importance of correctly specifying the strain rate when giving the yield stress of a metal. Not all metals exhibit a high strain-rate sensitivity. Aluminum and some of its alloys have either 0 or $-m$. In general, $m$ varies between 0.02 and 0.2 for homologous temperatures between 0 and 0.9 (90% of melting point in K). Therefore, one would have, at the most, an increase of 15% in the yield stress by doubling the strain rate. UNS G10400

Source: M.A. Meyers and K.K. Chawla, *Mechanical Metallurgy: Principles and Applications,* Prentice-Hall, 1984, p 572



## SS.055 310 annealed stainless steel sheet, effect of strain rate on mechanical properties

Sheet thickness = 1.60 mm (0.063 in.). Composition: Fe-25Cr-20.5Ni. UNS S31000

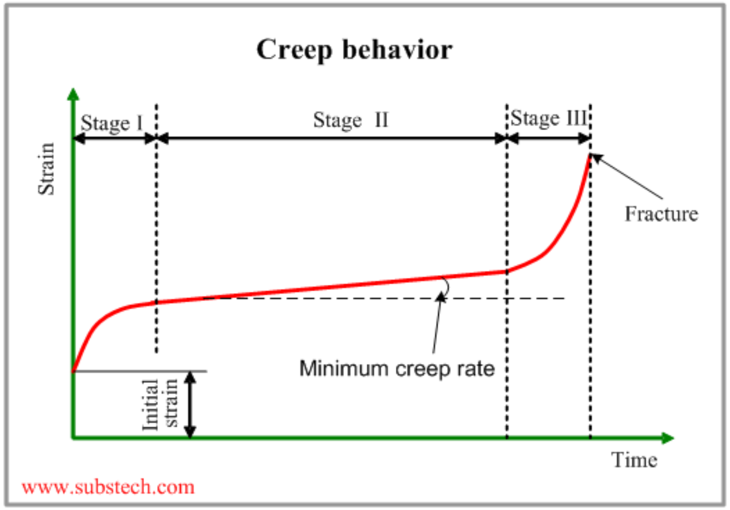
Source: R.G. Davies and C.L. Magee, The Effect of Strain-Rate upon the Tensile Deformation of Metals, *J. Eng. Mater. Technol.*, April 1975, p 151. As published in *Aerospace Structural Metals Handbook,* Vol 2, Code 1305, CINDAS/USAF CRDA Handbooks Operation, Purdue University, 1995, p 22

How do solids behave below their yield stress, especially when examined over very long time periods of stress?

Frost and Ashby found an answer to this question by analyzing the creep behavior of many types of materials under carefully controlled conditions

The answer: If brought within the range of measurability by increasing the test temperature, almost all materials will show creep, however low the applied stress

Creep is the steady-state, continuous deformation of a material when a stress below yield stress is applied, and this is usually after an initial linear elastic deformation and may take some time to arrive at steady state



The scientists took care to avoid the initiation and propagation of cracks by applying a very large hydrostatic pressure to seal up any opening cracks and observed pure creep behavior

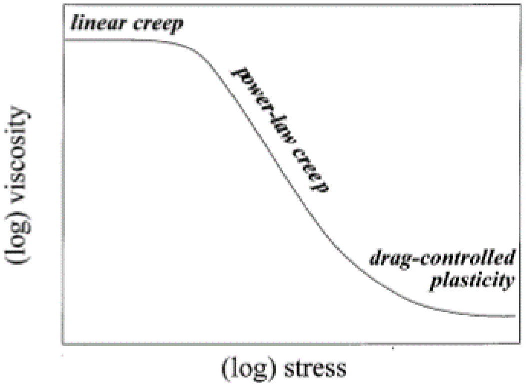
Pressurized specimen was taken to a high-enough temperature ($T>0.5T_m$)

At these temperatures atomic diffusion occurs and grains grow but the material is still solid

At T>0.5$T_m$ for all materials creep rates are brought within the measurable range, i.e. $10^{-9}$ $s^{-1}$

At these conditions all solids undergo creep and display three identical characteristics:

1. After a long-enough time, a steady creep rate is achieved
2. This creep rate, at low-enough applied stress, usually becomes a linear function of applied stress (Newtonian flow)—and is governed by atomic diffusion
3. At higher stresses, the creep rate increases faster than the stress, and usually shows a power-law-like non-Newtonian behavior



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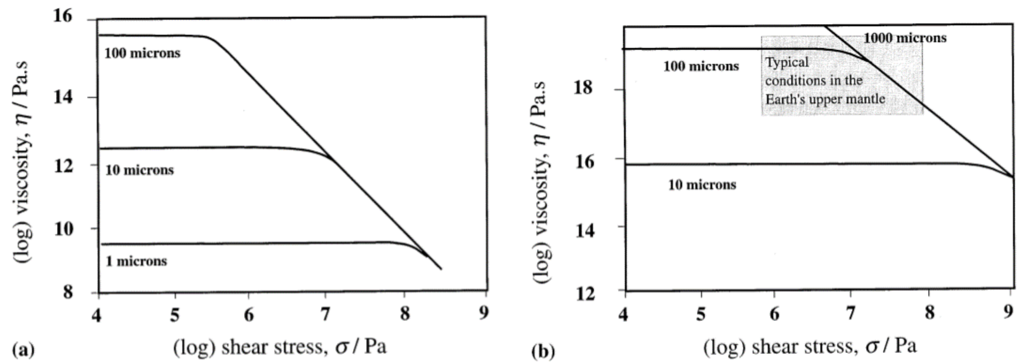

Fig. 18. Viscosity versus stress data at 1000°C, derived from [109], for various grain sizes of: (a) magnesium oxide; and (b) olivine (a rock).

Approximate values of the Ellis model parameters that describe the creep of some 'solid' materials

| Material | Grain size | Temp. | $\eta_0$ | $\sigma_0$ | $m$ |
|---|---|---|---|---|---|
| Spinel, a mineral | 10 | 1000 | $4.64 \times 10^{16}$ | $7.4 \times 10^{7}$ | 1.2 |
| α-Alumina | 10 | 1200 | $1.25 \times 10^{16}$ | $4.8 \times 10^{7}$ | 1.1 |
| Zinc carbide | 10 | 1600 | $2.00 \times 10^{16}$ | $1.5 \times 10^{6}$ | 2.9 |
| Lithium fluoride | 50 | 500 | $2.45 \times 10^{14}$ | $2.7 \times 10^{6}$ | 4.6 |
| Sodium chloride | 50 | 400 | $3.02 \times 10^{14}$ | $1.7 \times 10^{6}$ | 4.3 |
| 304 Stainless steel | 100 | 600 | $6.20 \times 10^{15}$ | $2.5 \times 10^{7}$ | 3.6 |
| Titanium | 100 | 900 | $1.00 \times 10^{15}$ | $1.0 \times 10^{6}$ | 2.9 |
| Ice | 100 | −8 | $1.02 \times 10^{13}$ | $3.2 \times 10^{5}$ | 2.1 |

$\eta_0$, the zero shear rate viscosity (Pa s); $\sigma_0$, the critical stress (Pa); and $m$, the non-linearity index [-]. The grain size (μm) and test temperature data (°C) are also included. The data has been read off graphs in Frost and Ashby's book [109] and has been fitted to the Ellis model. The grain size is included because it controls the values of $\eta_0$ and $\sigma_0$ (but not the value of $m$), Fig. 18.

# Creep of plastic materials

Some materials deform extensively above their yield strengths. Plastics refer to these materials with low yield strengths.

A common example is plasticine or play-doh. It is a mixture of clay and mineral oil.

This material deforms easily and to a large extent. It is often used to model metal working operations such as forging, rolling, milling, extrusion, and cutting

Its response to plastic deformation is represented as $\sigma = \sigma_Y + \mu\dot{\gamma}^n$

where $\sigma_Y$ is the yield stress

The creep behavior of plasticine was investigated by Barnes et al. under very long-time testing (days)

Its flow behavior under yield stress was seen as typical non-Newtonian, steady-state liquid behaviour

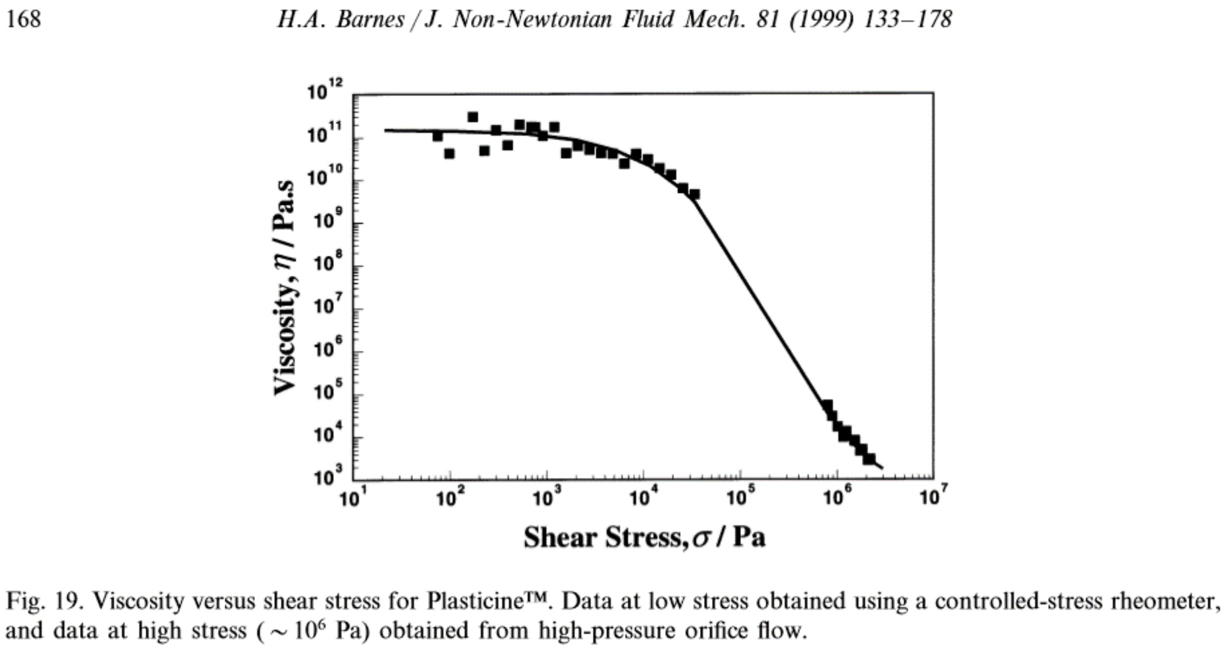

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Fig. 19. Viscosity versus shear stress for Plasticine™. Data at low stress obtained using a controlled-stress rheometer, and data at high stress ( $\sim 10^6$ Pa) obtained from high-pressure orifice flow.

This means that materials do not really have yield strengths when tested under precise rheometrical conditions, where very small shear rates are capable of being measured at very small stresses at very long times.

Yield strength is assigned to ductile solids for engineering purposes depending on measurements obtained under limited conditions.

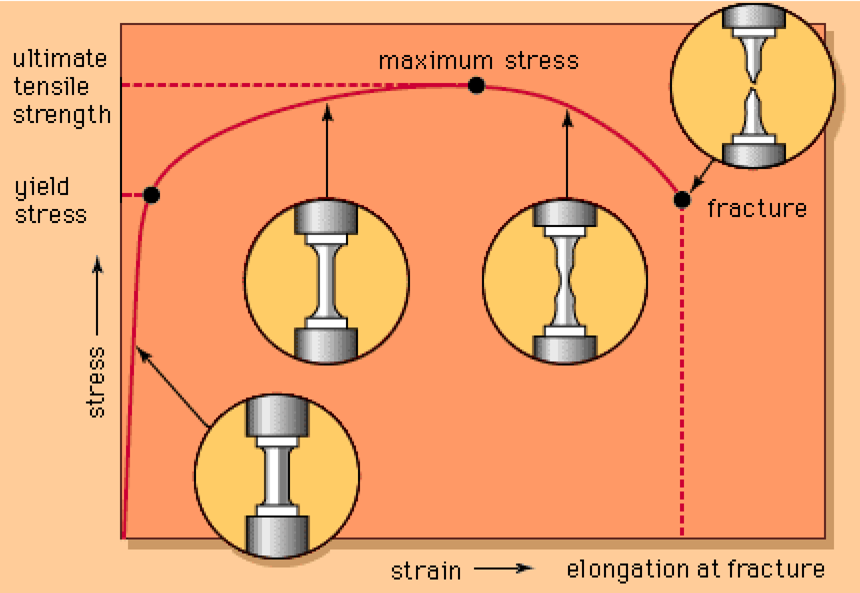
Plasticity

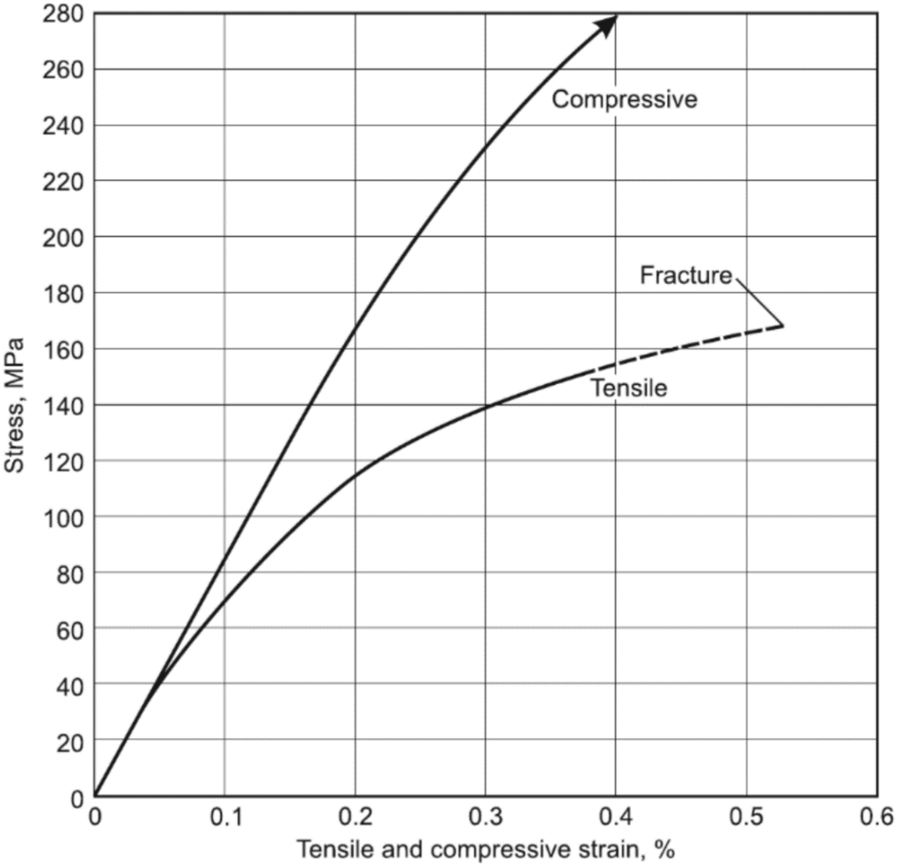
Just as the mechanical properties of a viscoelastic material vary with temperature and strain rate, so do plasticity.

Brittle failure is characterized by low strain and rupture that occurs at the highest stress reached.

Ductile failure involves plastic deformation after yielding, before rupture

Yielded materials exhibit necking and cold drawing. Necking occurs when the cross section is reduced abruptly. After necking material continues to extend, with the molecules reorienting themselves in the necked region at about constant force. This process of cold drawing produces a material, with its molecules now in a preferred orientation, that is much stiffer.

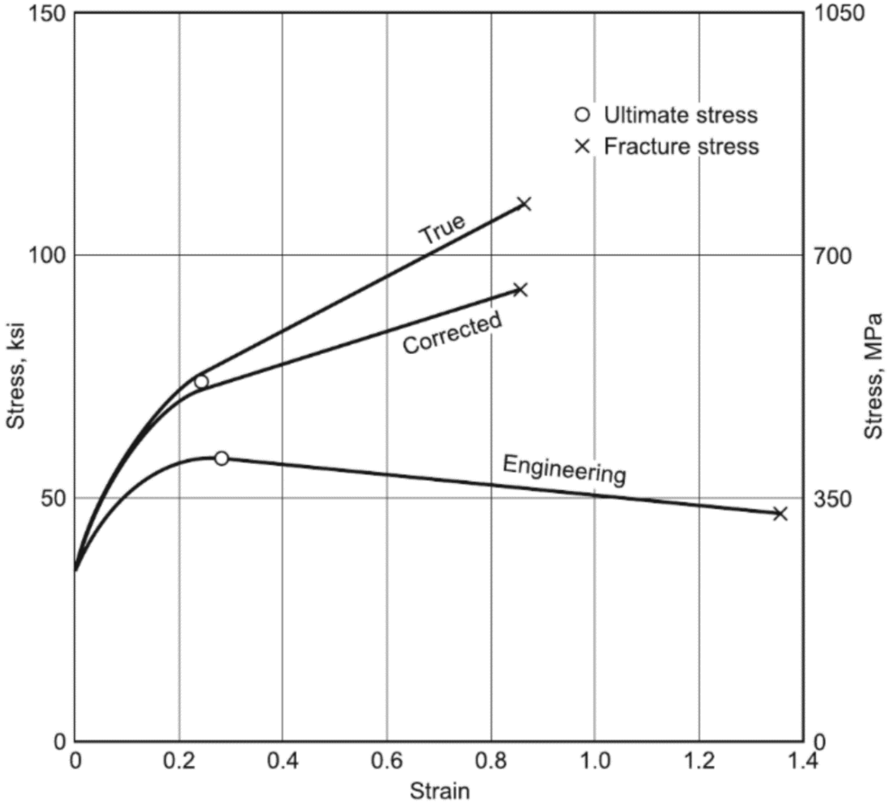




## CI.053 Flake graphite, gray iron casting, comparison of tensile and compressive stress-strain curves

Compressive strength $\simeq 600$ MPa

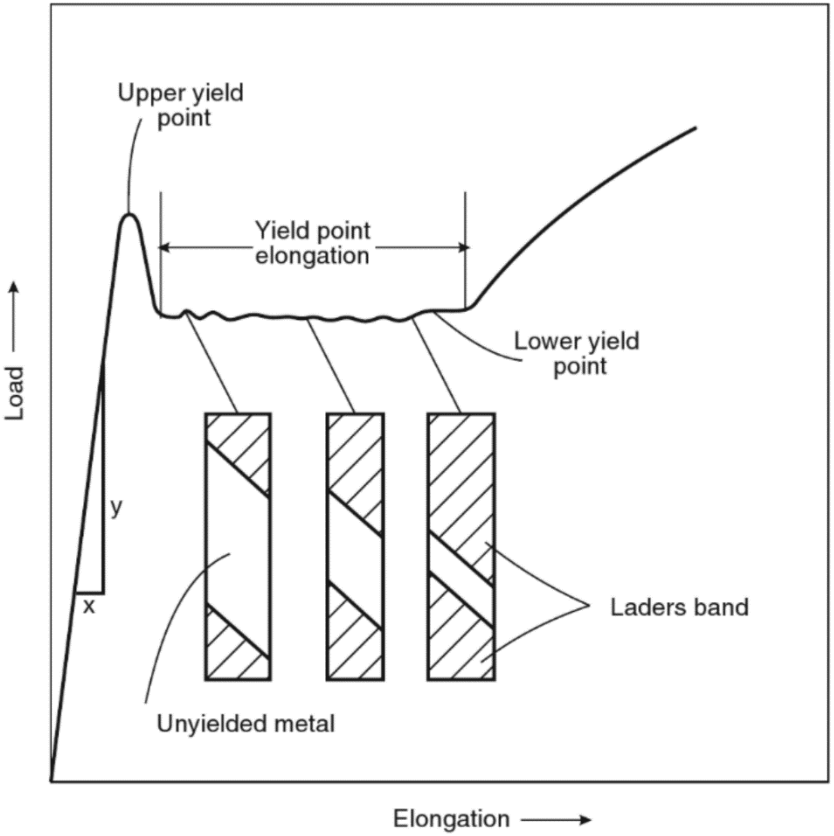
Source: “Stress/Strain Behaviour of Flake Graphite Cast Irons,” Broadsheet 157-1, British Cast Iron Research Association (BCIRA), 1977



## CS.039 1112 carbon steel, relationship of engineering, true, and corrected stress-strain curves

Relationship between engineering, true, and corrected tensile stress-strain curves for AISI 1112 steel. The figure above shows the relationship between the so-called engineering stress-strain curve based on the original area, the true stress-strain curve, and the corrected true stress-strain curve where the stress plotted ($\sigma_c$) is the uniaxial tensile stress in the absence of the hydrostatic component introduced by curvature of the neck. It is evident that interpretation of tensile test results is really quite involved despite the apparent simplicity of the test.

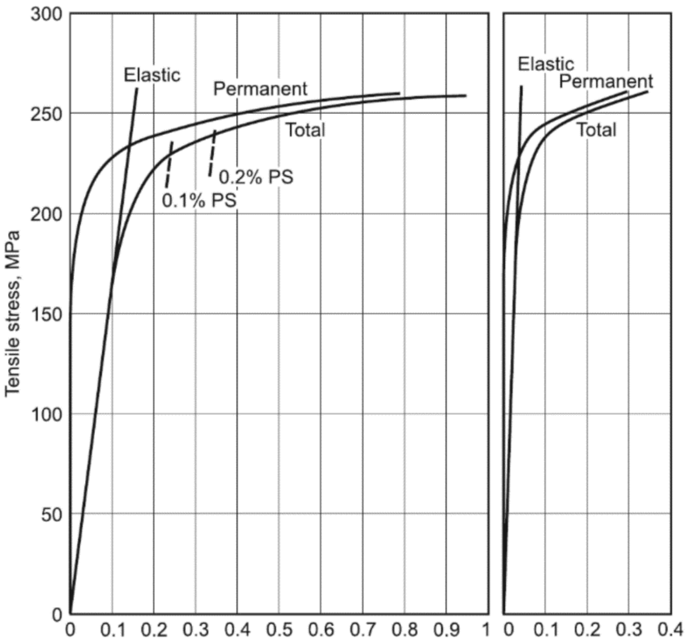
Source: M.C. Shaw, *Metal Cutting Principles,* Clarendon Press, Oxford, 1984, p 67



## CS.001 Annealed low-carbon steel, load-elongation curve showing Lüders bands

Typical yield point behavior of low-carbon steel. The slope of the initial linear portion of the stress-strain curve ($E = y/x$) is the modulus of elasticity. Many metals, particularly annealed low-carbon steel, show a localized, heterogeneous type of transition from elastic to plastic deformation that produces a yield point rather than a curve with a gradual transition from elastic to plastic behavior. The load increases steadily with elastic strain, then drops suddenly. After the upper yield point, several discrete bands of deformed metal, called Lüders bands, appear at stress concentrations, usually at about 45° to the tensile axis. Load fluctuates about some approximately constant value, and then rises with further strain.

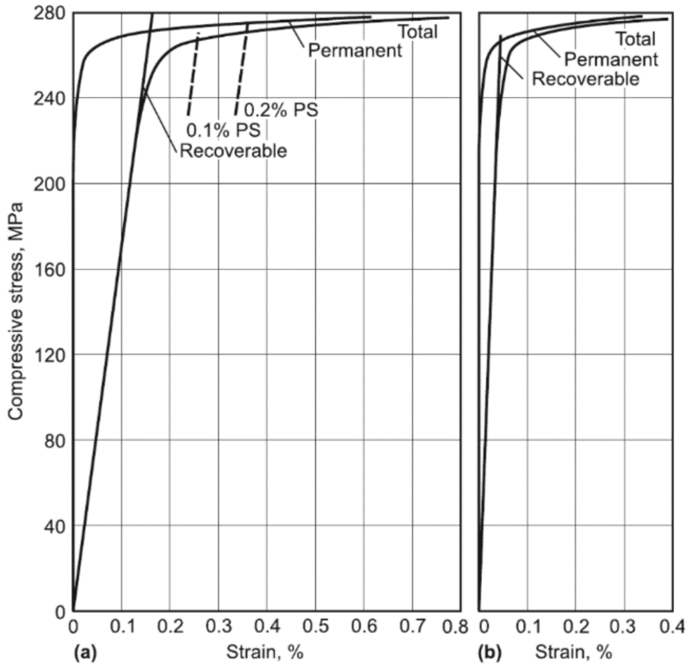
Source: G.E. Dieter, Mechanical Behavior under Tensile and Compressive Loads, *Mechanical Testing and Evaluation,* Vol 8, *ASM Handbook,* ASM International, 2000, p 100



### CI.025 Ferritic ductile iron casting, longitudinal tensile stress-strain curves (a) with lateral contraction (b)

Test specimen size = 28.651 mm diam × 76.2 mm gage length (1.128 in. diam × 3 in. gage length). Permanent strain remains when sample unloaded. Total strain is permanent plus recoverable. 0.1% proof stress (PS) = 232 MPa; 0.2% proof stress = 242 MPa. Composition: Fe-3.66C-1.8Si-0.41Mn-0.012S-0.025P-0.76Ni-0.064Mg

Source: G.N.J. Gilbert, The Stress/Strain Properties of Nodular Cast Irons in Tension and Compression, *BCIRA J.*, Vol 12 (No. 2), March 1964, p 177



### CI.026 Ferritic ductile iron casting, longitudinal compressive stress-strain curves (a) with lateral expansion (b)

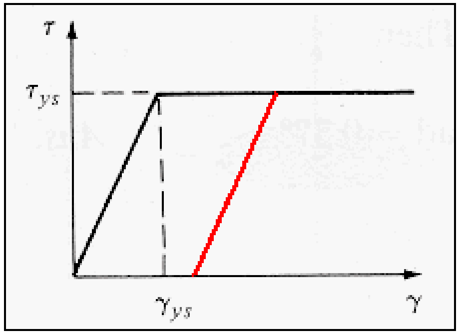
Test specimen size = 28.651 mm diam × 76.2 mm gage length (1.128 in. diam × 3 in. gage length). Permanent strain remains when sample unloaded. Total strain is permanent plus recoverable. 0.1% proof stress (PS) = 266 MPa; 0.2% proof stress = 267 MPa. Composition: Fe-3.66C-1.8Si-0.41Mn-0.012S-0.025P-0.76Ni-0.064Mg

Source: G.N.J. Gilbert, The Stress/Strain Properties of Nodular Cast Irons in Tension and Compression, *BCIRA J.*, Vol 12 (No. 2), March 1964, p 182

# Plastic deformation

The deformation and failure of plastic materials are hard to characterize because of the non-linear stress-strain relationships and changes in the structure of the material during deformation

The plastic behavior can be understood by considering an idealized elastoplastic material which is close to mild steel



When stresses reach the yield strength of the material, it starts yielding and keeps deforming plastically under a constant load

Unloading takes place along a straight line parallel to the elastic region when the load is removed

The segment in the horizontal axis represents the strain corresponding to the permanent deformation.

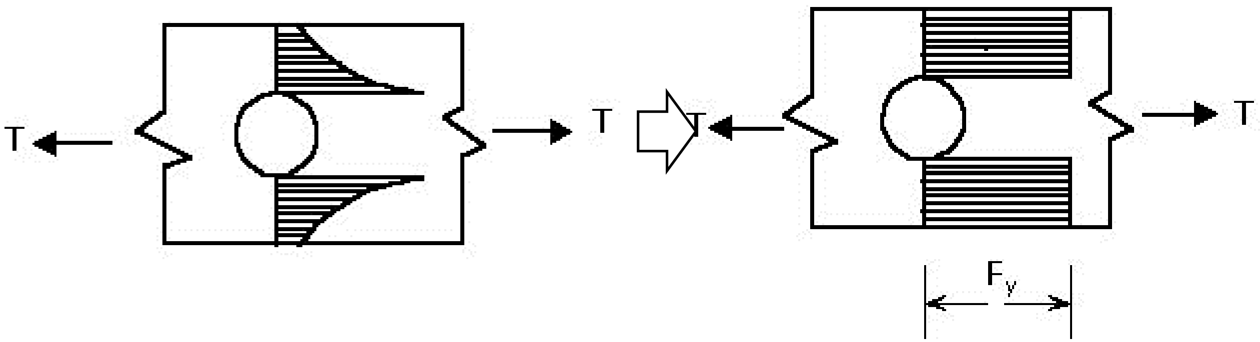
The distribution of stresses on a surface during plastic deformation of an elastoplastic material is as follows:

The area under the stress distribution curve represents the load applied. So the area and the value of maximum stress should increase as the load increases.

As the load is increased beyond the yield stress, the stress distribution curve flattens in the vicinity of a void since the stress in the material exceeded the value of yield strength.

The plastic zone where yield takes place keeps expanding as the load is further increased, until it reaches the edges of the plate.

At that point the distribution of stresses across the material is uniform at yield stress and the load is the largest which may be applied to the bar without causing rupture.



A mathematical theory on the fracture behavior plastic materials is not present since it usually occurs at high strains, which are mathematically difficult to characterize

Fracture mechanics of brittle materials as described by Griffith serves as a reference to understand the failure of plastics

According to Griffith, a fracture results in the formation of two new surfaces on each side of the crack and that the formation of these surfaces requires energy. This energy is stored as changes in bond length throughout the rest of the material as it is stretched

Fracture then involves the transmission of this energy to the fracture surfaces, at the same time relaxing the strain in the area from which energy has been released

Propagation of the crack depends on the crack length and the critical length should be exceeded for the energy transmitted by the surrounding material to cause brittle fracture.

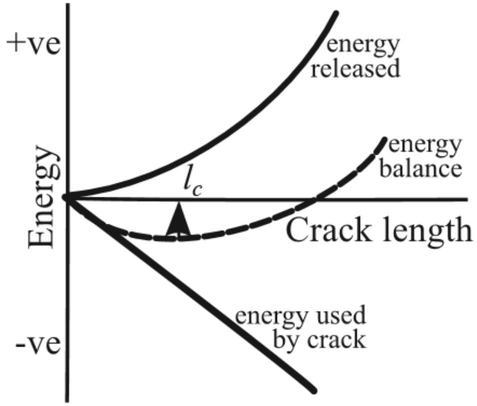


Figure 1.21. The energy conditions associated with the propagation of cracks showing the derivation of the critical crack length, *lc*. The term "+ve" indicates strain energy released by the crack; "–ve" indicates energy used to make new surfaces as the crack progresses.

But energy in plastic material can be used up before it can reach the crack tip to contribute to the new fracture surface

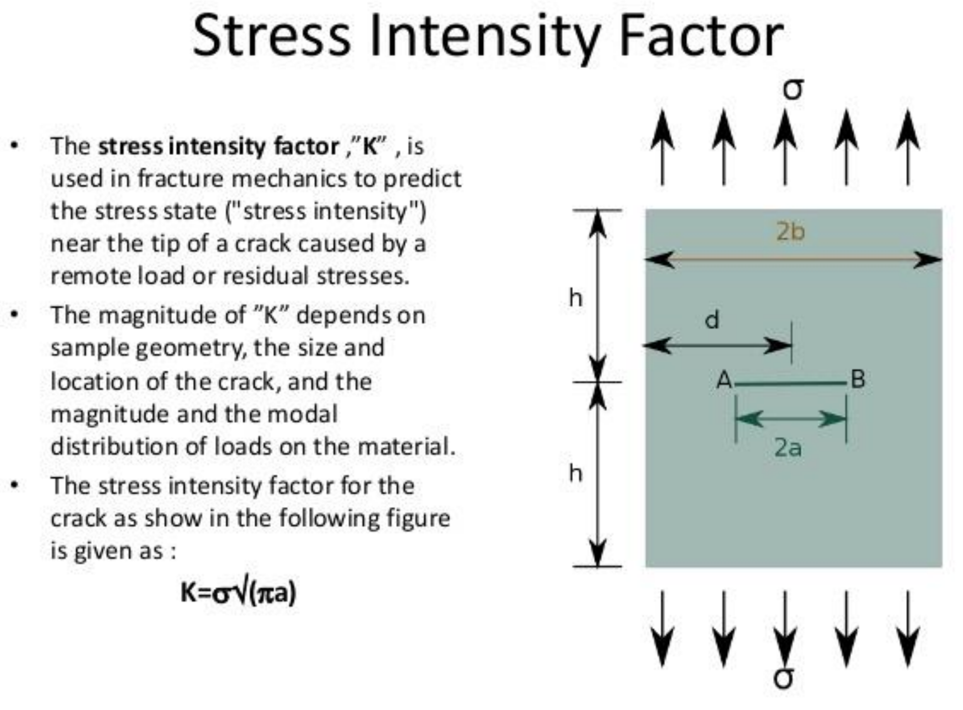
Ductility of metals can account for far more energy than is needed for propagation of the crack

Based on Griffith’s theory, two general criteria need to be satisfied for a piece of material to fracture:

- The strength of the material (the critical stress intensity, KC) must be reached
- There must be sufficient elastic strain energy (work of fracture) available at the tip of the crack to propagate it.

In engineering, the critical stress intensity is known as the fracture toughness

Toughness of a material is important to resist the initiation of a crack.



## Stress Intensity Factor

- The **stress intensity factor** ,”**K**” , is used in fracture mechanics to predict the stress state ("stress intensity") near the tip of a crack caused by a remote load or residual stresses.
- The magnitude of ”K” depends on sample geometry, the size and location of the crack, and the magnitude and the modal distribution of loads on the material.
- The stress intensity factor for the crack as show in the following figure is given as :

$$\mathbf{K=\sigma\sqrt{(\pi a)}}$$