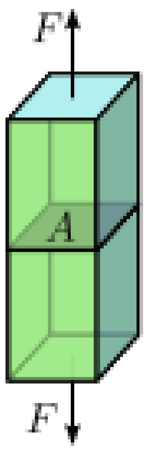
Plasticity and Deformation Process

The general relationships between stress and strain

in elastic deformation

Both the stresses and the deformations of a material under stress should be analyzed to completely understand its deformation behavior

Let’s consider deformation of plates under axial stresses



The normal stress is equal to the load per area of surface perpendicular to the load

$$\sigma = \frac{F}{A}$$

The deformation of the material per unit length is the normal strain, ε

$$\varepsilon = \frac{\delta}{L}$$

The stress-strain diagram helps us determine the modulus of elasticity of the material, whether it is ductile or brittle and whether the strains in the plate will disappear when the load is removed

The stress-strain diagram of a material is obtained by conducting a tensile test on the specimen of material

The length and the cross-sectional area are recorded as the load is increased at a constant rate

A ductile material elongates linearly at a very slow rate as the load is increased

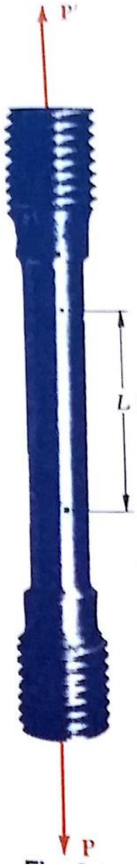
After yield stress is reached the specimen undergoes a large deformation with a relatively small increase in the applied load.

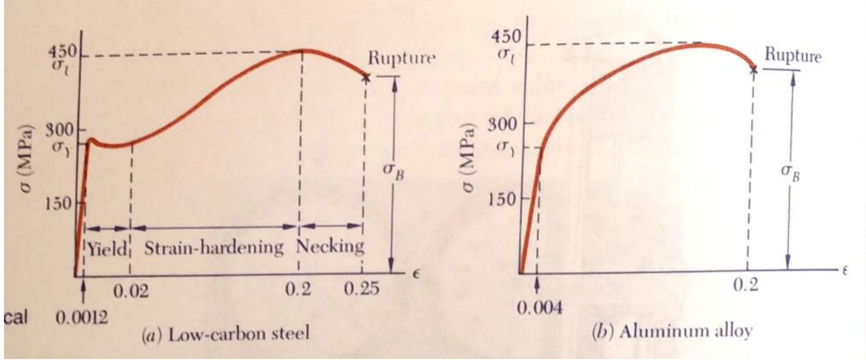
Yielding is caused by slippage of the material along oblique crystal surfaces and is primarily due to shearing stresses.

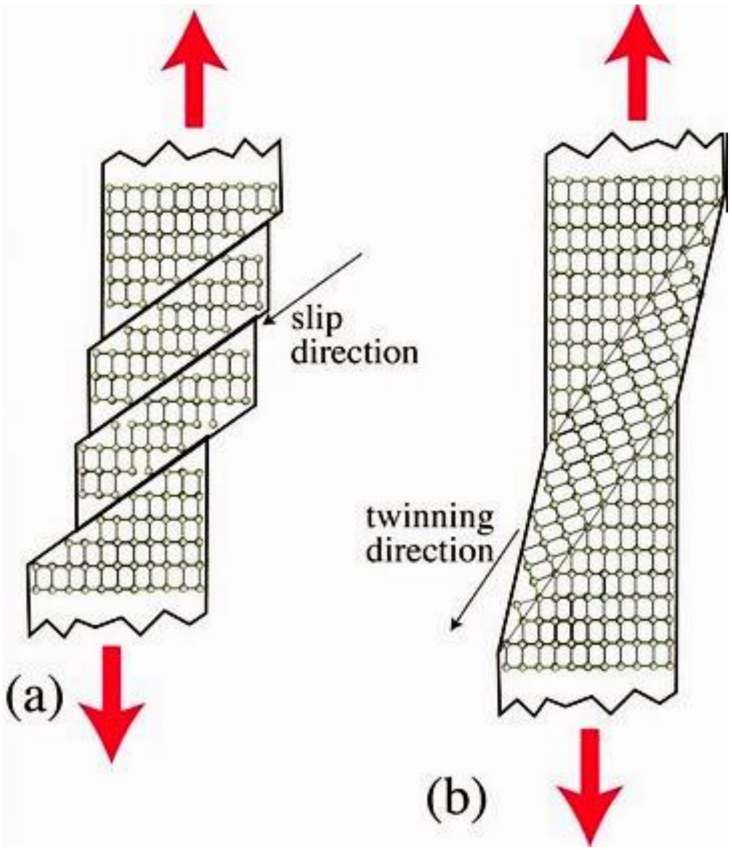
The diameter of a portion of the specimen begins to decrease due to local instability and results in necking

Lower loads are sufficient to rupture the specimen after necking

Rupture of ductile materials occurs along a cone-shaped surface which forms an angle of approximately 45 with the normal surface which shows that shear is primary casue of rupture.






slip
direction
twinning
direction
(a)
(b)

In axial stress, value of σ obtained as $\frac{F}{A}$ represents the average stress over the section rather than the stress at a specific point P. Note that the stresses are uniform only when the direction of the load passes through the centroid of the material.

The stress at point P is defined as

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

The stress value at point P is generally different than the average stress and is found to vary across the section. This variation is small in any section away from the points of application of the loads

In practice, the distribution of the normal stress in an axially loaded material is assumed to be uniform, except in the immediate vicinity of the points of application of the loads.

Consider the case when two rigid plates are used to transmit loads to the material
The plates move towards each other, causing the material to get shorter and increase in width and thickness, the distribution of stresses is uniform throughout the material and $\sigma_y = \left(\sigma_y\right)_{ave} = \frac{P}{A}$

Consider now that the load is concentrated
The material in the immediate vicinity of the points of application of the load are subjected to very large stresses
Other parts of the material away from the ends, at a distance equal to or greater than the width of the material are unaffected and load is distributed uniformly

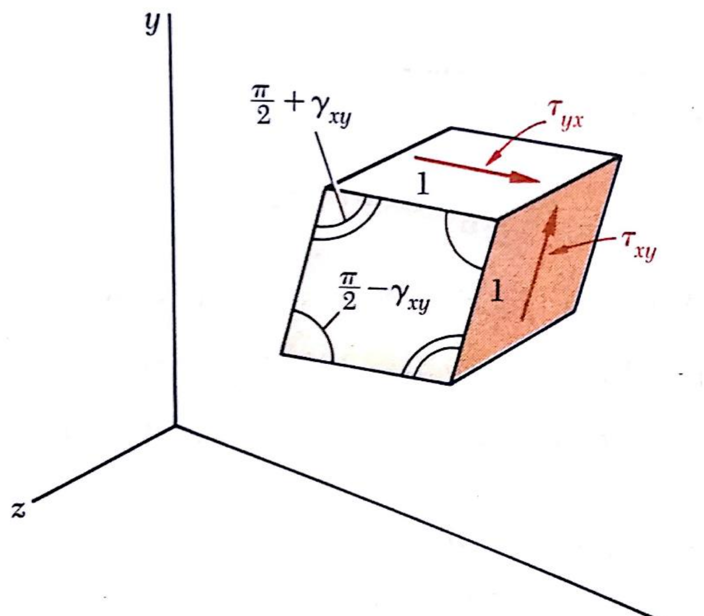
Shearing stresses are created when transverse forces are applied to a material.

These stresses vary greatly across the section and their distribution cannot be assumed uniform

The average shearing stress over the section is obtained by dividing the shearing load by the cross-sectional area:

$$\tau_{ave} = \frac{F}{A}$$

The deformation of a three dimensional structure under transverse loads is represented as the shear strain which is the change in angle of the initially perpendicular normal and transverse surfaces.



Both normal and shearing stresses develop in a material subjected to axial loading.

If we consider an oblique section inside the material under axial stress, the normal and shear stresses are related to the angle θ formed by the section with a normal plane:

$$\sigma = \frac{F}{A_0}\cos^2\theta$$

$$\tau = \frac{F}{A_0}\sin\theta\cos\theta$$

where A_0 is the area of a section perpendicular to the axis of the material

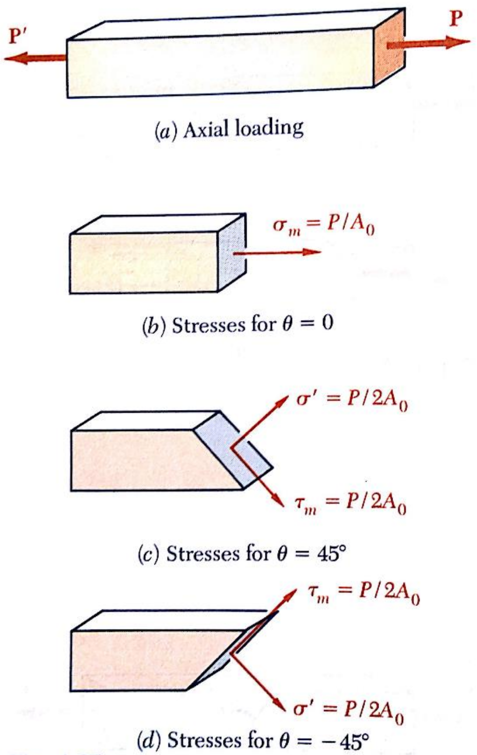


Fig. 1.27

Note that the normal stress is maximum and equal to $\frac{F}{A_0}$ for θ=0, while the shear stress is maximum and equal to $\frac{F}{2A_0}$ for θ=45

Most structural materials and machine components are under more complex loading conditions than axial and transverse loadings

For a point Q inside a three dimensional body subjected to various loads in various directions, the stress condition created by the loads can be determined.

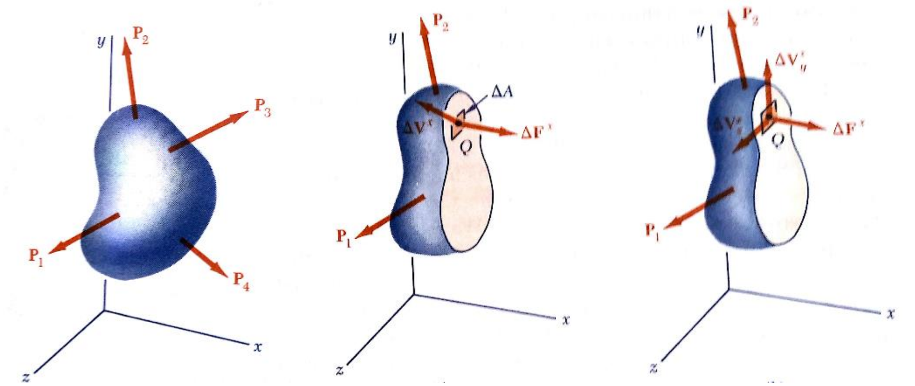
Sectioning the body at point Q using a plane parallel to the yz plane will help us visualize the stress conditions at the plane and point of Q.

The normal and shear stresses on point Q are calculated as

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V^x{}_y}{\Delta A}$$

$$\tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V^x{}_z}{\Delta A}$$



where ΔF^x is the normal force, ΔV^x is the shearing force acting on the surface perpendicular to the x-axis, τ_{xy} is the shear stress perpendicular the x-axis, parallel to the y-axis and τ_{xz} is the shear stress perpendicular the x-axis, parallel to the z-axis.

While the normal force ΔF^x has a well-defined direction, the shearing force ΔV^x may have any direction in the plane of the section. So it is resolved into two component forces $\Delta V^x{}_y$, $\Delta V^x{}_z$ in direction parallel to the y and z axes.

When that analysis is applied to the portion of body located to the right of the vertical plane through Q, the same magnitudes but opposite directions are obtained for the normal and shearing forces ΔF^x,$\Delta V^x{}_y$, $\Delta V^x{}_z$

This time the section faces the negative x axis, so a positive sign for σ_x indicates that the corresponding arrow points in the negative z direction and negative y and z directions for τ_{xy}, τ_{xz}

The same analyses can be done parallel to the zx plane to obtain the stress components σ_y, τ_{yz}, τ_{yx} and to the xy plane to obtain components σ_z, τ_{zx}, τ_{zy}

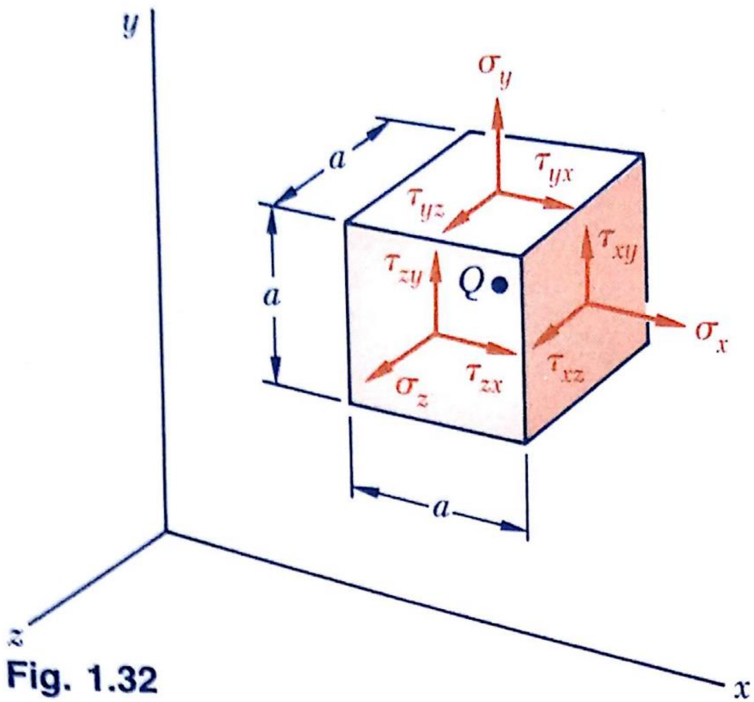


Fig. 1.32

Poisson's ratio

The normal stresses acting on the three faces of an axially loaded specimen of material are

$$\sigma_x = E\epsilon_x, \sigma_y = 0, \sigma_z = 0$$

But the corresponding normal strains are

$$\epsilon_x = {}^{\sigma_x}/_{E}, \epsilon_y = \epsilon_z < 0$$

because an elongation produced by an axial tensile force is accompanied by a contraction in any transverse direction in all isotropic, homogeneous materials

The absolute value of the ratio of the lateral strain to the axial strain is the Poisson's ratio

$$\upsilon = \left|\frac{\epsilon_y}{\epsilon_x}\right| = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$

So

$$\epsilon_x = {}^{\sigma_x}/_{E}, \epsilon_y = \epsilon_z - {}^{\sigma_x \upsilon}/_{E}$$

The volume of the material also changes as a result of axial elongation and transverse contraction

Multiaxial stress

The stress condition at point Q is understood clearly by considering a small cube of side a, centered at Q and the stresses exerted on each face of the cube.

Only three faces are visible but equal and opposite stress components act on the faces at the back.

As cube length a gets smaller, the error involved in the difference between stresses at the face and point is minimized. If we consider the point Q as a small cube we can calculate the normal and shearing forces acting on the various faces with area ΔA.

The condition for mechanical equilibrium for an object is that all of the forces and moments acting on three axes add up to zero:

$$\sum F_x = 0 \,, \sum F_y = 0 \,, \sum F_z = 0$$

$$\sum M_x = 0 \,, \sum M_y = 0 \,, \sum M_z = 0$$

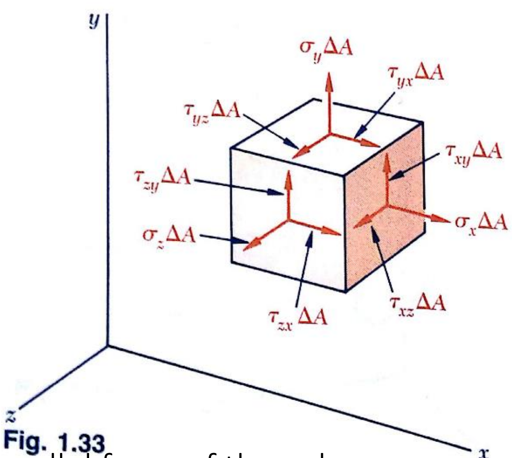


Fig. 1.33

$\sum F_{x,y,z} = 0$ are satisfied since equal and opposite forces are acting on parallel faces of the cube

$\sum M_z = 0$ is satisfied because the shear forces rotating the object around the z-axis are opposite, one causing counterclockwise and the other clockwise rotation:

$$\sum M_z = (\tau_{xy}\Delta A)a - (\tau_{xy}\Delta A)a = 0$$

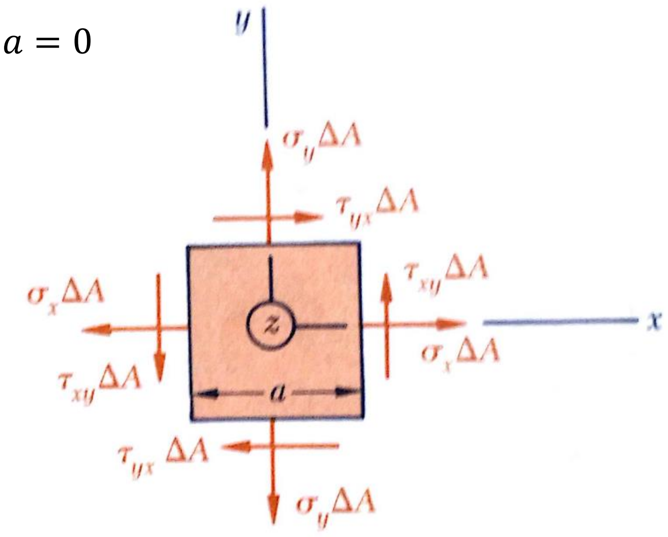


Fig. 1.34

And we conclude that

$$\tau_{xy} = \tau_{yx}$$

Similarly from the remaining equations $\sum M_x = 0$, $\sum M_y = 0$, the relations between other shear stresses are obtained:

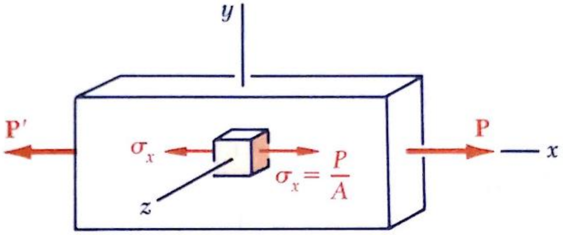
$$\tau_{yz} = \tau_{zy} \quad \tau_{zx} = \tau_{xz}$$

Hence it is seen that only 6 stress components are required to define the condition of stress at a given point Q

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Shear stresses are present in all stressed materials except hydrostatically stressed ones

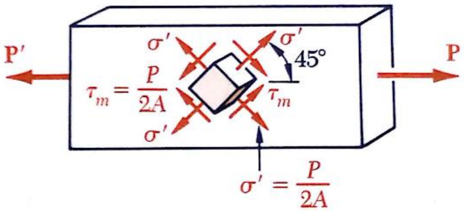
Consider the case of axial loading. The conditions of stress in part of the material is described as:



The only stresses acting on the small cube parallel to the loaded surface are normal stresses because

$$\sigma = \frac{P}{A_0}\cos^2 0 = \frac{P}{A_0}$$
$$\tau = \frac{P}{A_0}\sin 0 \cos 0 = 0$$

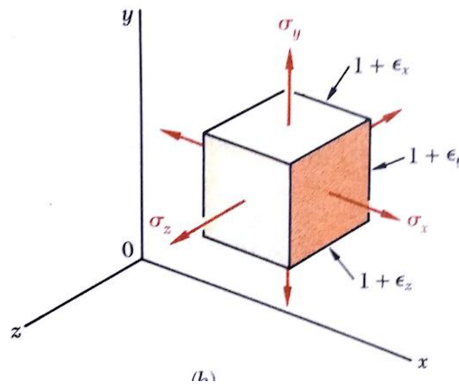
However if the small cube is rotated by 45 around z-axis, normal and shearing stresses of equal magnitude are exerted on four faces of the cube



The same loading condition may lead to different deformations at different points in a body, depending on the orientation of the element considered.

In the case of multiaxial loading of a material, deformations and the strains can be calculated by dividing the loads into normal and shearing forces in the three axis.

Consider the material as a unit cube with sides equal to 1. Consider also only the normal components of stress to calculate the normal strains as a generalized Hooke's law



The effect of each normal stress component on the strain components ϵ_x, ϵ_y, ϵ_z should be calculated separately and combined.

$$\epsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

This result is valid only as long the stresses do not exceed the proportional limit and as long as the deformations involved are small

To consider the shear strains in a real stress situation, the shearing stress components should also be considered

Shearing stresses tend to deform a cubic material into an oblique parallelepiped

Two of the angles formed by the four faces under stress are reduced from $\frac{\pi}{2}$ to $\frac{\pi}{2} - \gamma_{xy}$ and two angles are increased from $\frac{\pi}{2}$ to $\frac{\pi}{2} + \gamma_{xy}$

The angle γ_{xy} is the shear strain corresponding to the x and y directions in the plane of z

Shear strains on other planes may also be calculated similarly

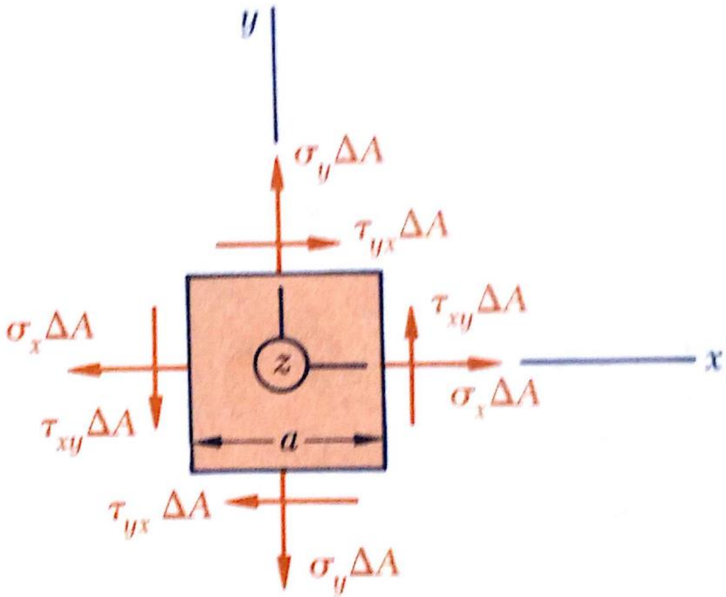
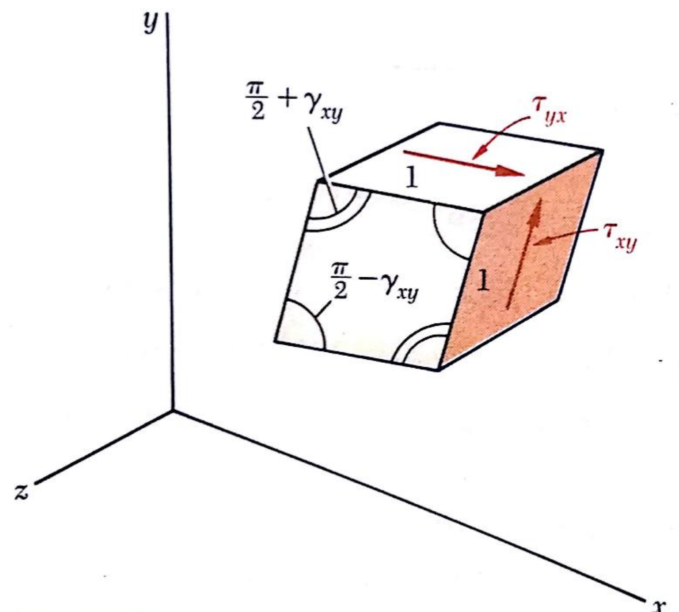


Fig. 1.34



Plotting successive values of τ_{xy} against γ_{xy} gives us the shearing stress-strain diagram for the material

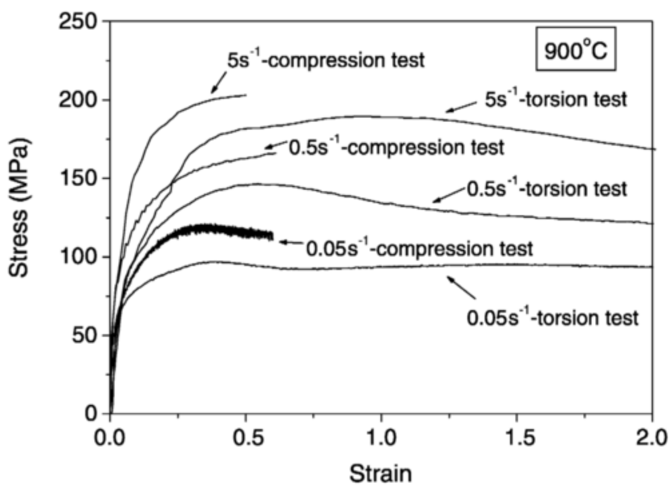
Obtained yield strength, ultimate strength values are about half of the material from the tensile test

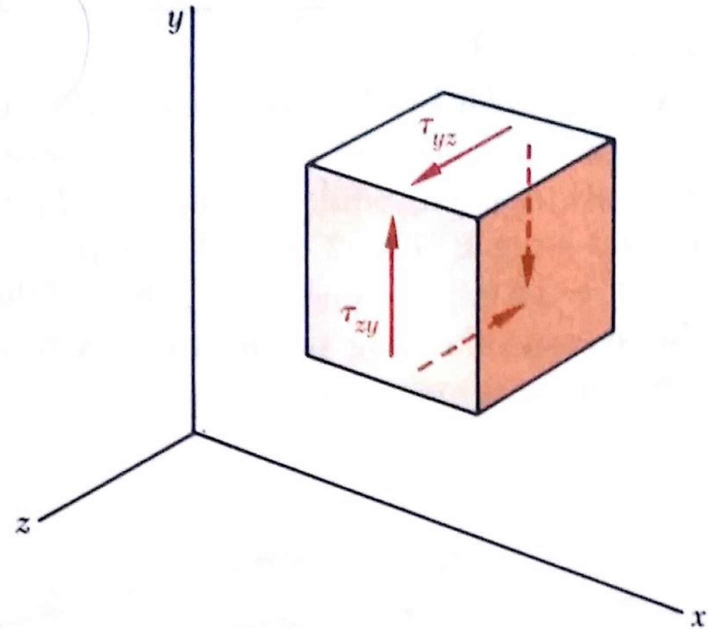
For values of the shearing stress which do not exceed the proportional limit in shear, Hooke's law for shearing is defined as

$$\tau_{xy} = G\gamma_{xy}$$

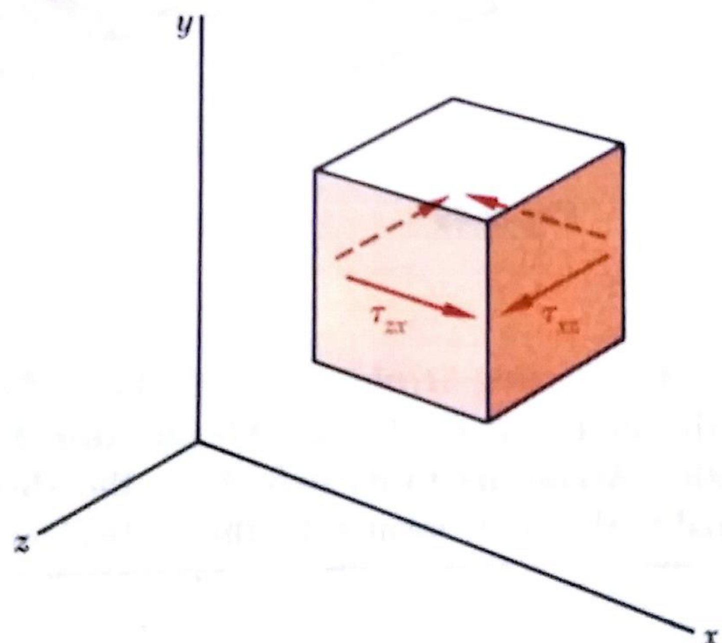
The relations for the other planes are obtained by considering the corresponding stress components

$$\tau_{yz} = G\gamma_{yz} \qquad\qquad \tau_{zx} = G\gamma_{zx}$$





(a)



The following group of equations representing the generalized Hooke's law for a homogeneous isotropic material under multiaxial loading is obtained:

$$\epsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\tau_{xy} = G\gamma_{xy} \qquad \tau_{yz} = G\gamma_{yz} \qquad \tau_{zx} = G\gamma_{zx}$$

The 6 strain components of a material under stress in the elastic region can be calculated from the 6 stress components and any 2 of E, G, ν

$$\epsilon = \begin{bmatrix} +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} & \frac{\tau_{xy}}{G} & \frac{\tau_{xz}}{G} \\ \frac{\tau_{xy}}{G} & -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E} & \frac{\tau_{yz}}{G} \\ \frac{\tau_{xz}}{G} & \frac{\tau_{yz}}{G} & -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E} \end{bmatrix}$$

The strain energy of deformation per unit of a linear elastic material is

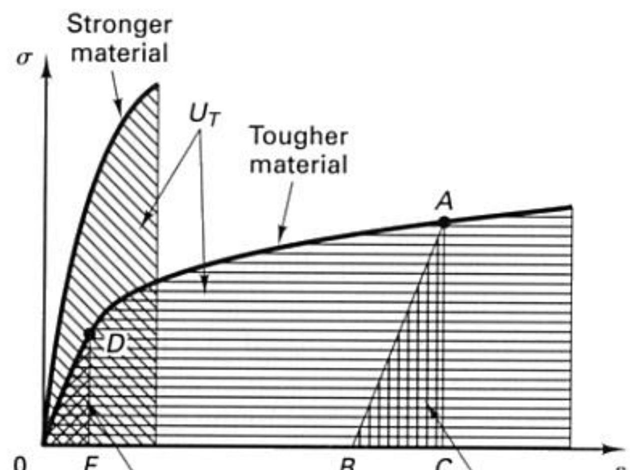
$$U_0 = \frac{1}{2}\sigma_{ij}\epsilon_{ij}$$

$$U_0 = \frac{1}{2}\left(\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + \sigma_{zz}\epsilon_{zz} + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx}\right)$$

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$$\epsilon = \begin{bmatrix} +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} & \frac{\tau_{xy}}{G} & \frac{\tau_{xz}}{G} \\ \frac{\tau_{xy}}{G} & -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E} & \frac{\tau_{yz}}{G} \\ \frac{\tau_{xz}}{G} & \frac{\tau_{yz}}{G} & -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E} \end{bmatrix}$$

$$U_0 = \frac{1}{2}\left(\sigma_x\left(\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}\right) + \sigma_y\left(-\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}\right) + \sigma_z\left(-\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}\right) + \tau_{xy}\frac{\tau_{xy}}{G} + \tau_{yz}\frac{\tau_{yz}}{G} + \tau_{xz}\frac{\tau_{xz}}{G}\right)$$

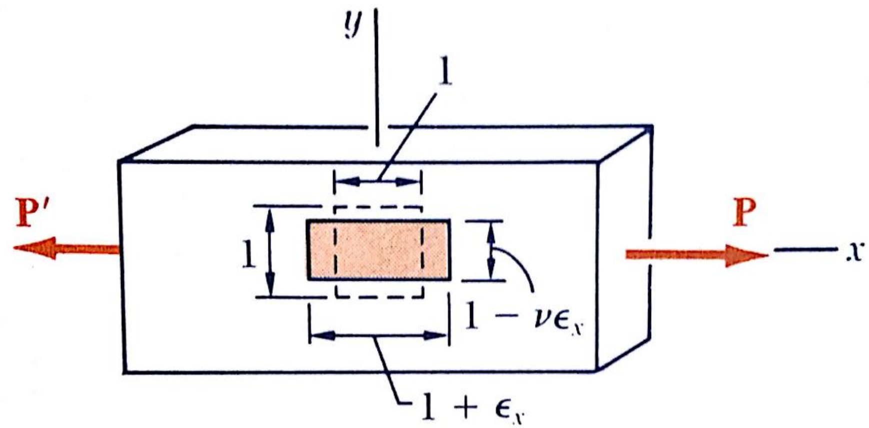


A material of rectangular prism geometry that is subjected to axial loading deforms into a rectangular parallelepiped. The prism will elongate along the axis of the tensile force and will contract in both of the transverse y and z directions:

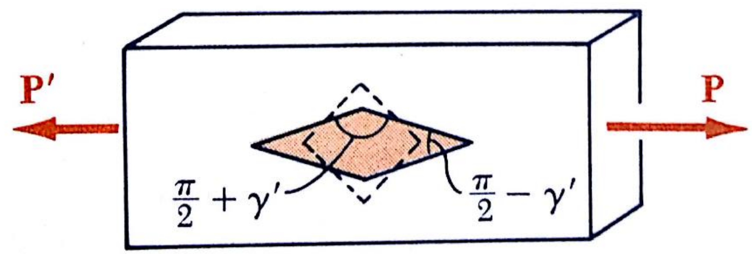
For an element in the shape of a cube with unit sides, the deformed geometry will have sides $1+\epsilon_x$, $1-\nu\epsilon_x$, $1-\nu\epsilon_x$ according to the Hooke's law

If the element is oriented at 45 to the axis of the load, the cube will transform into an oblique parallelepiped due to the shearing stress components:

Each of the right angles increases or decreases by the shearing strain γ'that is induced by the shear stress component of the axial stress and $\gamma' = \gamma_{max}$



(a)



(b)

Consider the prismatic element obtained by intersecting the unit cubic element by a diagonal plane

It deforms into a slice which has horizontal and vertical sides equal to $1+\epsilon_x$ and $1-\nu\epsilon_x$ by the shearing force:

The angles of the undeformed and the deformed slices are one half of $\frac{\pi}{2}$ and $\frac{\pi}{2}-\gamma_{max}$ respectively

$$\beta = \frac{\pi}{4} - \frac{\gamma_{max}}{2}$$

$$\tan\beta = \frac{\tan\frac{\pi}{4} - \tan\frac{\gamma_{max}}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\gamma_{max}}{2}} = \frac{1-\tan\frac{\gamma_{max}}{2}}{1+\tan\frac{\gamma_{max}}{2}}$$

Since $\frac{\gamma_{max}}{2}$ is a very small angle,

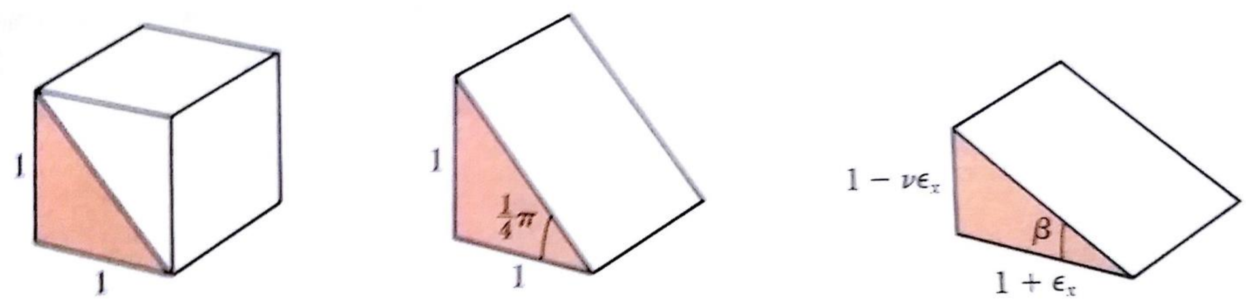
$$\tan\beta = \frac{1-\frac{\gamma_{max}}{2}}{1+\frac{\gamma_{max}}{2}}$$

Also

$$\tan\beta = \frac{1-\nu\epsilon_x}{1+\epsilon_x}$$

Hence,

$$\gamma_{max} = \frac{(1+\nu)\epsilon_x}{1+\frac{1-\nu}{2}\epsilon_x}$$



$$\gamma_{max} = \frac{(1+\nu)\epsilon_x}{1 + \frac{1-\nu}{2}\epsilon_x}$$

The denominator may be taken as 1 since $\epsilon_x \ll 1$, and the relation between the maximum shearing strain and the axial strain is obtained:

$$\gamma_{max} = (1+\nu)\epsilon_x$$

According to Hooke's law,

$$\gamma_{max} = \frac{\tau_{max}}{G} \qquad\qquad \epsilon_x = \frac{\sigma_x}{E}$$

$$\frac{\tau_{max}}{G} = (1+\nu)\frac{\sigma_x}{E}$$

$$\frac{E}{G} = (1+\nu)\frac{\sigma_x}{\tau_{max}}$$

Recall that the axial stress and the maximum shearing stress are defined as $\sigma_x = \frac{P}{A}$ and $\tau_{max} = \frac{P}{2A}$

$$\frac{E}{G} = 2(1+\nu)$$

or

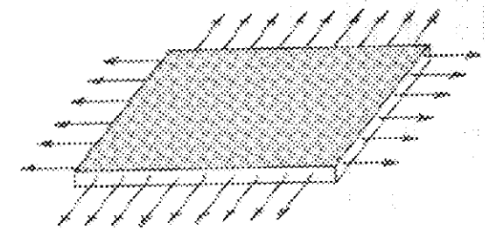
$$\frac{E}{2G} = (1+\nu)$$

Hence the constants E, G and ν are related to each other for any material

(a) Uniaxial stress : $\begin{pmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

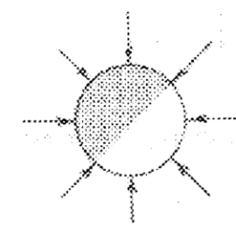
(a)

(b) Biaxial stress : $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$



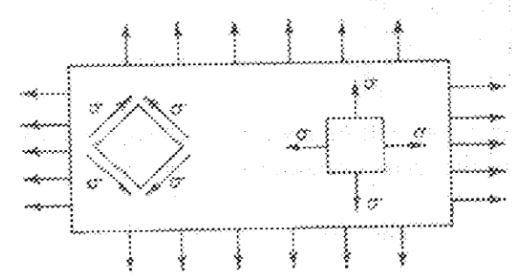
(b)

(c) Hydrostatic pressure : $\begin{pmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{pmatrix}$



(c)

(d) Pure shear : $\begin{pmatrix} 0 & \sigma & 0 \\ \sigma & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$



(d)

• Special Stress States •

Hydrostatic and Deviatoric Stresses

Total stress tensor can be divided into two components:

1. Hydrostatic or mean stress tensor (σ_m) involving only pure tension or compression
2. Deviatoric stress tensor (σ'_{ij}) representing pure shear with no normal components

$$\sigma_m = \frac{\sigma_{kk}}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$\sigma_{ij} = \sigma'_{ij} + \frac{1}{3}\delta_{ij}\sigma_{kk}$$

Example on Hydrostatic and Deviatoric Stresses

Given the stress state: $\sigma_{ij} = \begin{pmatrix} 80 & 20 & -50 \\ 20 & -40 & 30 \\ -50 & 30 & 50 \end{pmatrix}$,

a. *Find the hydrostatic part of the stresses.*

b. *Find the deviatoric part of the stresses.*

Ans. (a) $\sigma_{ij}^{hyd} = \sigma_m \delta_{ij} = \frac{1}{3}(\sigma_{11}+\sigma_{22}+\sigma_{33})\ \delta_{ij}$ where $\sigma_m = \frac{1}{3}(80 - 40 + 50) = 30$ so that

$$\sigma_{ij}^{hyd} = \begin{pmatrix} 30 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 30 \end{pmatrix}.$$

(b) By definition, $\sigma_{ij}^{dev} = \sigma_{ij} - \sigma_{ij}^{hyd} = \begin{pmatrix} 80 & 20 & -50 \\ 20 & -40 & 30 \\ -50 & 30 & 50 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 30 \end{pmatrix} = \begin{pmatrix} 50 & 20 & -50 \\ 20 & -70 & 30 \\ -50 & 30 & 20 \end{pmatrix}$

Note that the mean hydrostatic stress for $\sigma_{ii}^{dev} = (\sigma_{11}^{dev} + \sigma_{22}^{dev} + \sigma_{33}^{dev}) = 0$, as expected.

The stress-strain diagram of a material is obtained by conducting a tensile test on the specimen of material

The initial portion of the stress-strain diagram shows proportionality between the stress applied and the resultant strain according to the Hooke's law

$$\sigma = E\varepsilon$$

The largest value of the stress for which Hooke's law can be used for a material is the proportional limit

For ductile materials with a well defined yield point, it coincides with the yield point, for others Hooke's law can be used for stress values slightly larger than the proportional limit.

Physical properties of materials like strength, ductility, corrosion resistance may be significantly affected by alloying, heat treatment and manufacturing processes

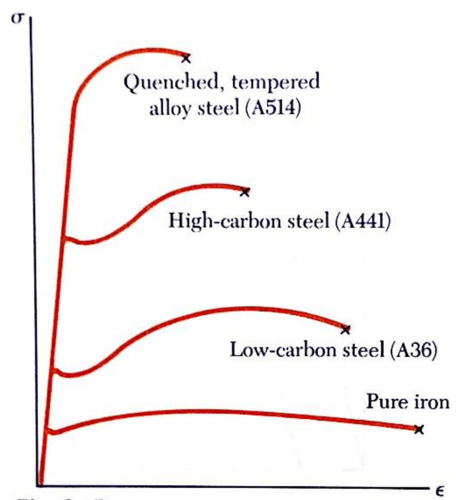
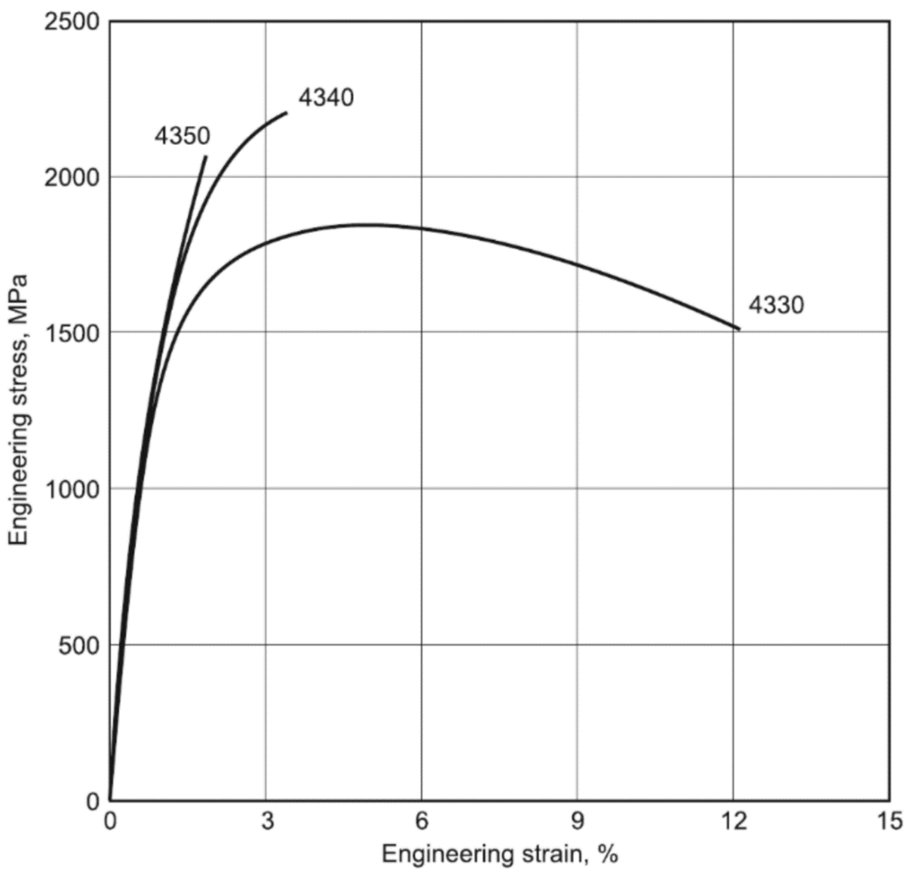


Fig. 2.15

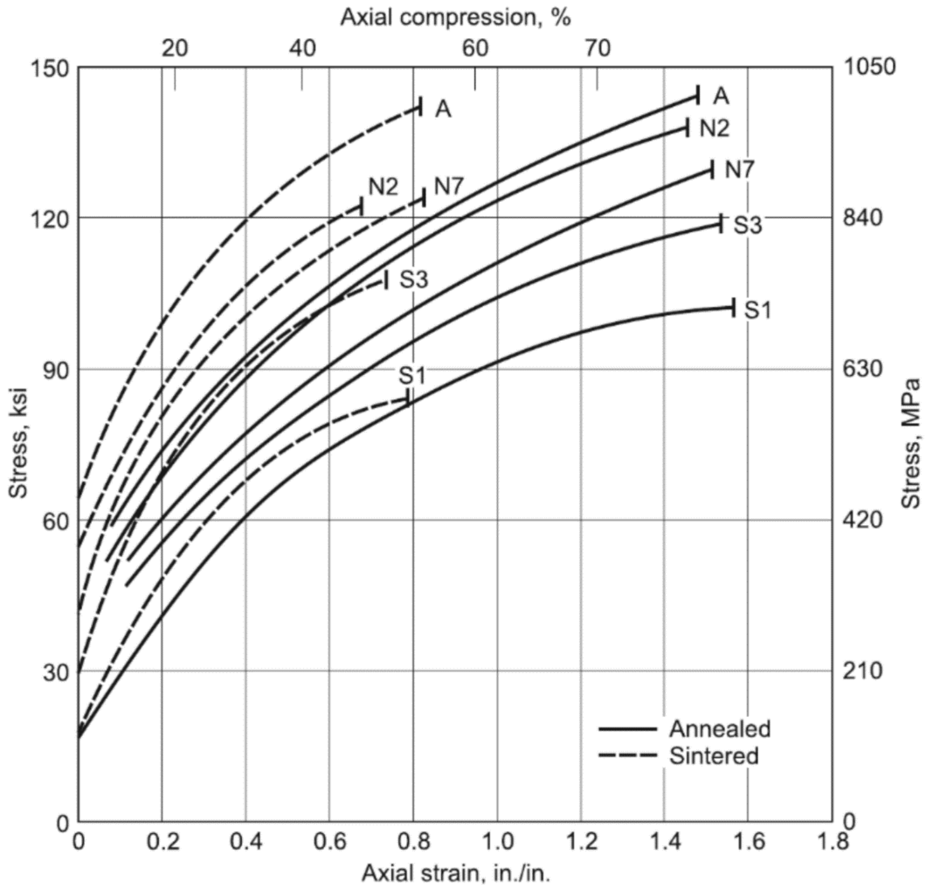
Although the variation of stress with strain for pure iron and different grades of steel is great and show different yield strength, ultimate strength and ductility, they posses the same stiffness



AS.019 4330, 4340, 4350 nickel-chromium-molybdenum alloy steel hot-rolled plate, tensile engineering stress-strain curves

Test direction: long transverse. Specimen size = 6.25 mm diam × 38 mm long, austenitized in salt bath at 936 °C, 20 min, oil quenched. Tested as-quenched with Instron machine with crosshead velocity of 8.5 mm/s, which corresponds to strain rate of 0.0033/s

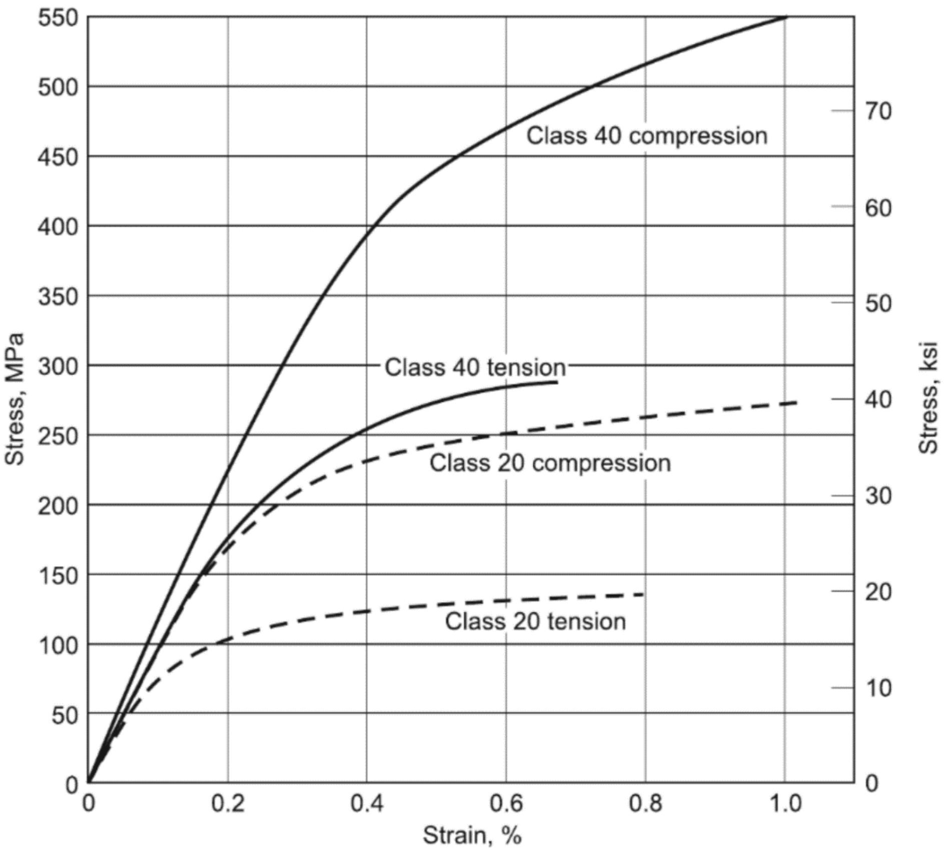
Source: M. Saeglitz and G. Krauss, Deformation, Fracture, and Mechanical Properties of Low-Temperature-Tempered Martensite in SAE 43xx Steels, *Metall. Mater. Trans.,* Vol 28A (No. 2), Feb 1997, p 382



CI.077 Steel preform powder metal forged cylinder, compressive stress-strain curves

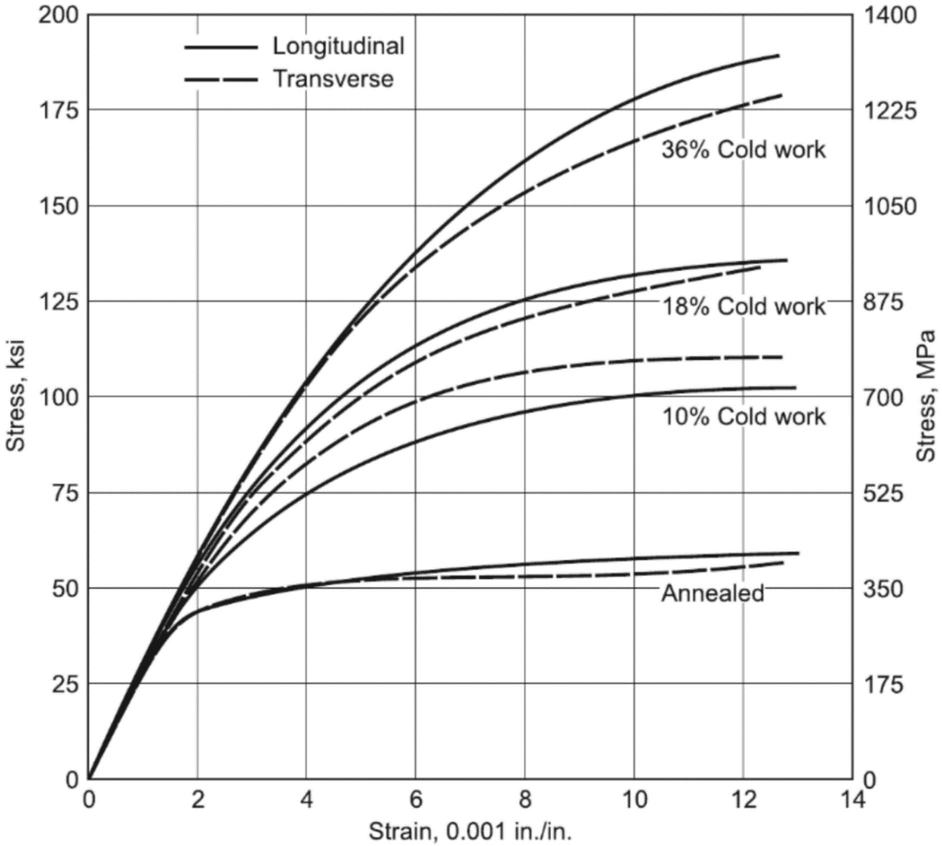
Test direction: longitudinal. Five steel powder compositions used: A, Fe-0.27C-2.0Ni-0.5Mo; N2, Fe-0.17C-2.7Ni-0.8Cr; N7, Fe-0.24C-0.6Ni-0.5Cr-0.2Mo; S1, Fe-0.01C; S3, Fe-0.33C. Preforms compacted to 785 MPa (114 ksi), sintered at 1199 °C (2190 °F), 30 min, and spheroidized (heating three times above and below eutectoid point). The sintered and annealed preforms are compared.

Source: *Source Book on Cold Forming,* American Society for Metals, 1975, p 208



CI.048 Class 20 and 40 gray iron casting, tensile and compressive stress-strain curves

Source: J.L. Herron, R.A. Flinn, and P.K. Trojan, Research for the article: Mechanical Properties of Gray Iron, *Iron Castings Handbook,* C.F. Walton, Ed., Iron Casting Society, 1981, p 235



SS.001 201 stainless steel, stress-strain curves showing effect of cold work

Test direction: longitudinal and transverse. Composition: Fe-17Cr-6.5Mn-4.5Ni. UNS S20100

Source: P.D. Harvey, *Engineering Properties of Steel,* American Society for Metals, 1982

If the strains caused in a material by the application of an axial load disappear when the load is removed, the material behaves elastically. The largest value of stress for which elastic behavior continues is the elastic limit.

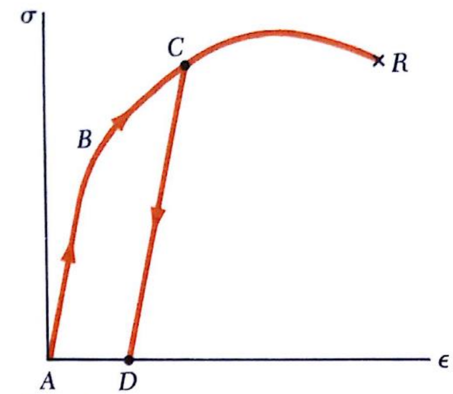


Fig. 2.16

The elastic limit, proportional limit and the yield point are equal for plastic materials with a well-defined yield point

The material behaves elastically below the yield point

Above the yield point the material yields. When the load is removed the stress and strain decrease linearly along a line parallel to the elastic loading curve.

Overall strain does not return to zero because plastic deformation has taken place above the yield stress.

The plastic deformation depends not only upon the maximum value reached by the stress but also upon the time elapsed before load is removed.

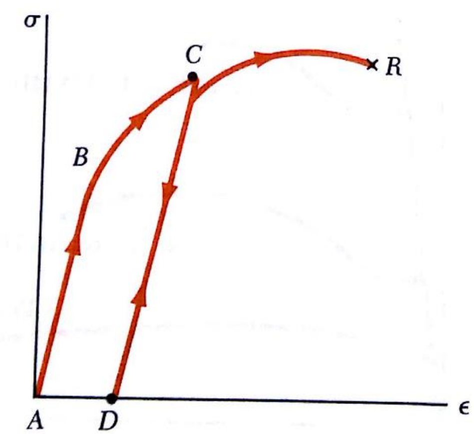


Fig. 2.17

If after being loaded and unloaded, the material is loaded again, the new curve will rise parallel to the initial curve until it almost reaches point C and then connect with the curved portion of the original diagram.

Note that the straight elastic portion of the new loading curve is longer than the initial due to strain hardening.

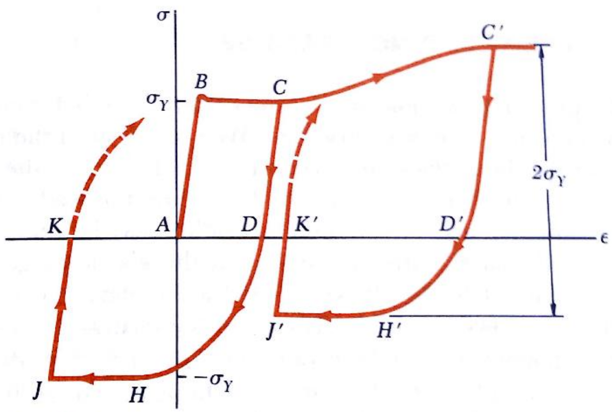
The elastic limit has increased but the ductility decreased since the point of rupture is not changed.

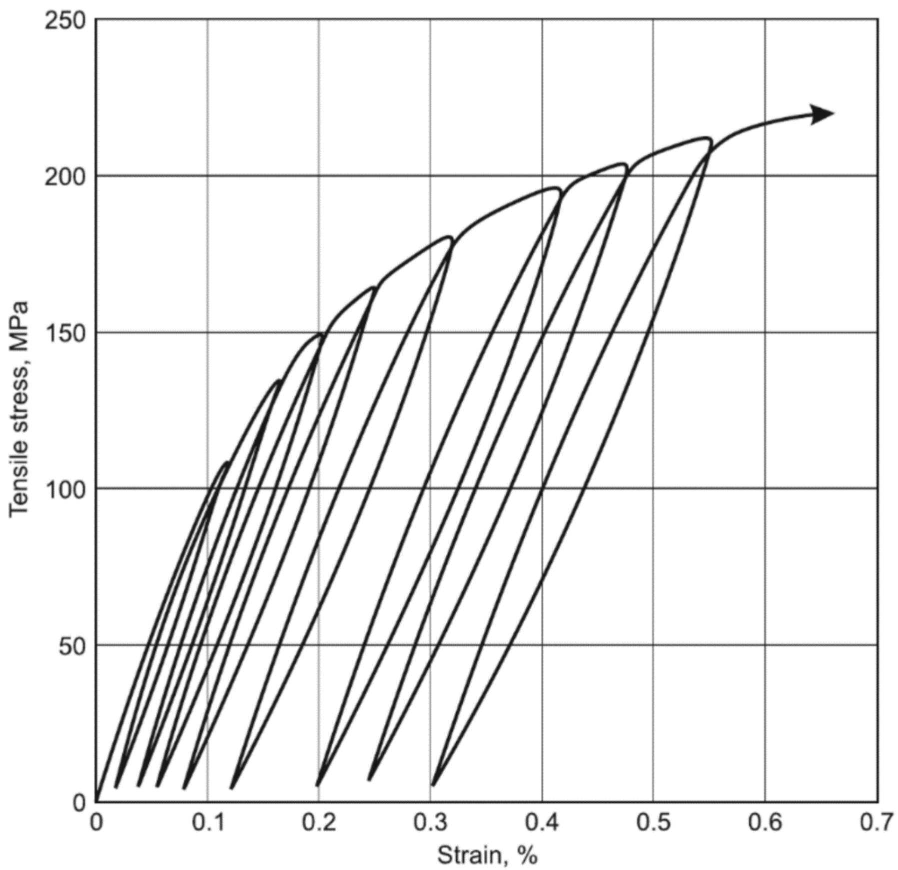
If the second loading was done in compression instead of tension, an interesting behavior is observed for ductile materials

A compressive load is applied after the initial tensile load is removed at point D, compressing the material until the negative yield strength along a curved portion DHJ of the diagram

When the compressive load is removed at point J the stress returns to zero with an equal slope to the elastic region.

If the initial tension is large enough to cause strain hardening until point C', the second compressive stress reaches its maximum value at H' where material yields. While the maximum value of the compressive stress is less than the yield stress, the total change is equal to twice the yield strength of the material.

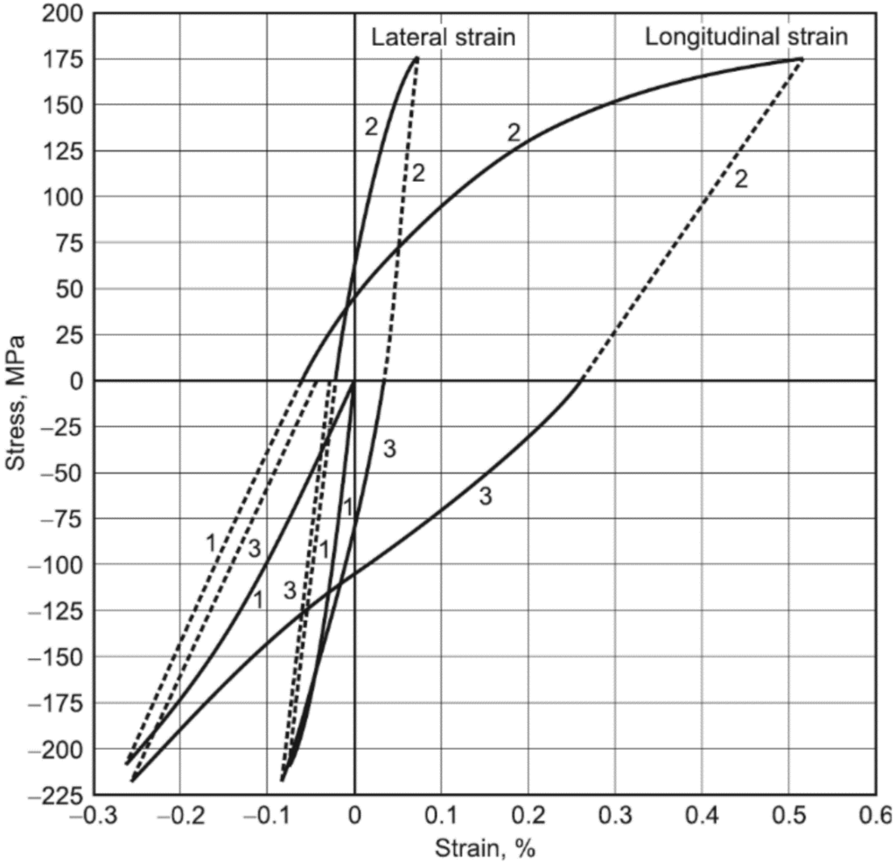




CI.052 Flake graphite, gray iron casting, tensile stress-strain curves with cyclic loading to increasing stress levels

Ultimate strength = 230 MPa. Permanent deformation increases with increasing stress levels.

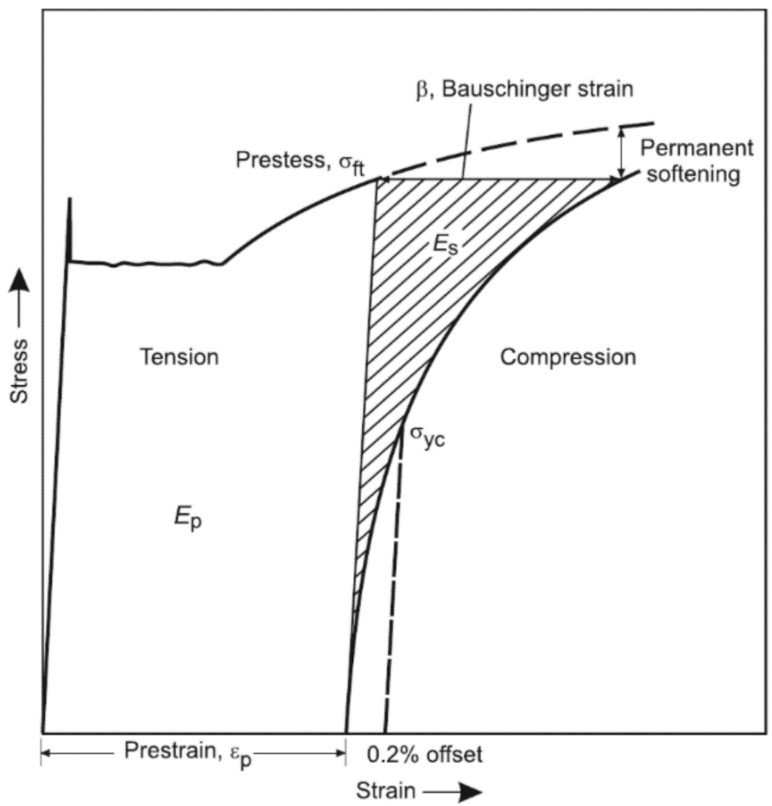
Source: "Stress/Strain Behaviour of Flake Graphite Cast Irons," Broadsheet 157-1, British Cast Iron Research Association (BCIRA), 1977



CI.051 Gray iron casting, tensile and compressive longitudinal and lateral stress-strain curves

Progression of test follows numbers 1–3 (solid line 1 to dashed line 1 to solid line 2 to dashed line 2, etc.). Solid lines are load applications; dashed lines are relaxations. These are relatively high stresses. Composition: Fe-3.2C-2.19Si-0.56Mn-0.031S-0.046P

Source: G.N.J. Gilbert, Stress/Strain Properties of Cast Iron and Poisson's Ratio in Tension and Compression, *BCIRA J.*, Vol 9 (No. 3), May 1961, p 351



CS.025 Carbon steel, Bauschinger effect on stress-strain curves

The elastic limit of a metal is lowered after reverse loading. The area E_p is the energy expended in prestrain, and E_s is the energy saved in reverse loading.

Source: C.-C. Li, J.D. Flasck, J.A. Yaker, and W.C. Leslie, On Minimizing the Bauschinger Effect in Steels by Dynamic Strain Aging, *Metall. Trans. A,* Jan 1978, p 86

Fatigue

Although the elastic region of the stress-strain curve is a safe zone against plastic deformation, repeating the load many times (millions) will cause rupture at a stress much lower than the yield strength of the material, hence fatigue.

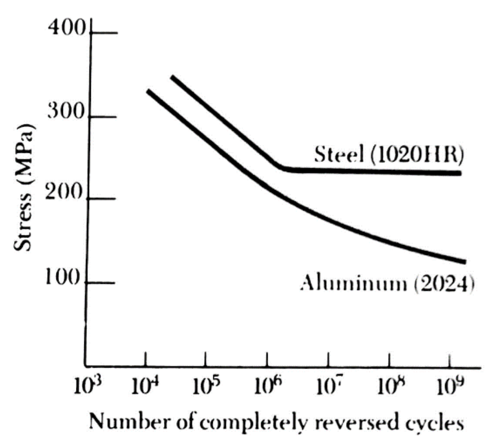
The stress-cycle curve for steel shows that relatively few cycles are enough to cause rupture if the applied maximum stress is high.

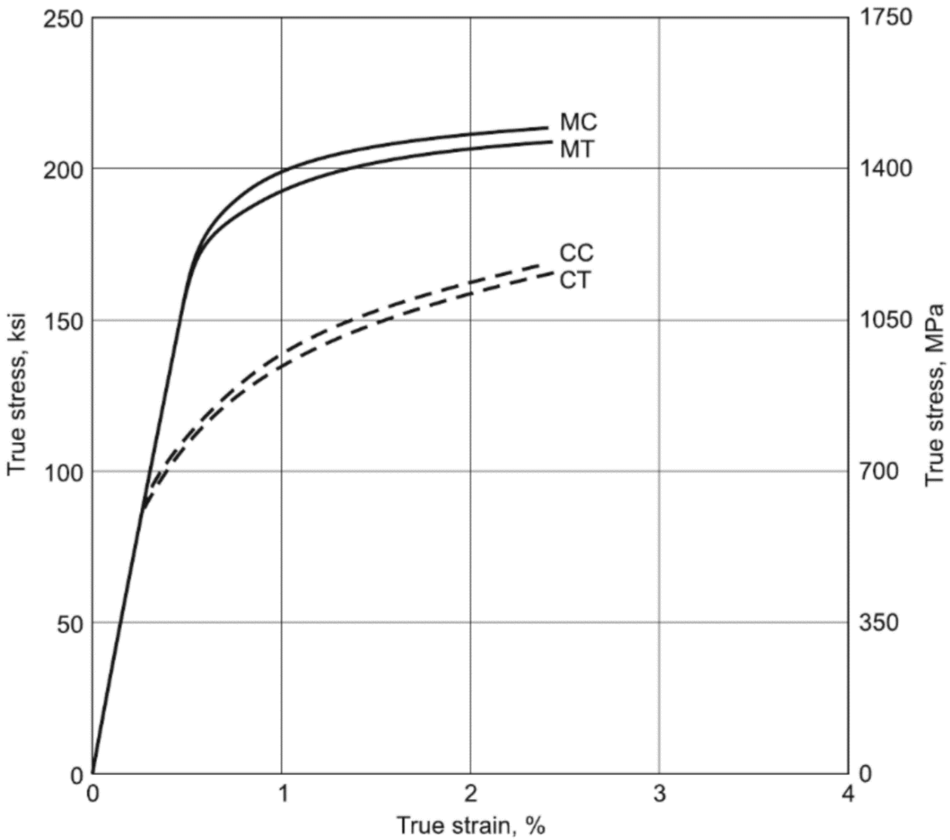
As the magnitude of stress is reduced, the number of cycles to rupture increases. For steel a stress limit is reached called the endurance limit, below which it will not rupture even for infinitely many cycles.

For non-ferrous metals like aluminum, the stress at failure continues to decrease as the number of loading cycles is increased. The stress around 500 million cycles is called the fatigue limit.

The mechanism of fatigue is slow propagation of a crack that initiate at an imperfection with each cycle

Sudden brittle fracture occurs when the amount of undamaged material is insufficient to carry the maximum load
Figure fatigue fracture surface

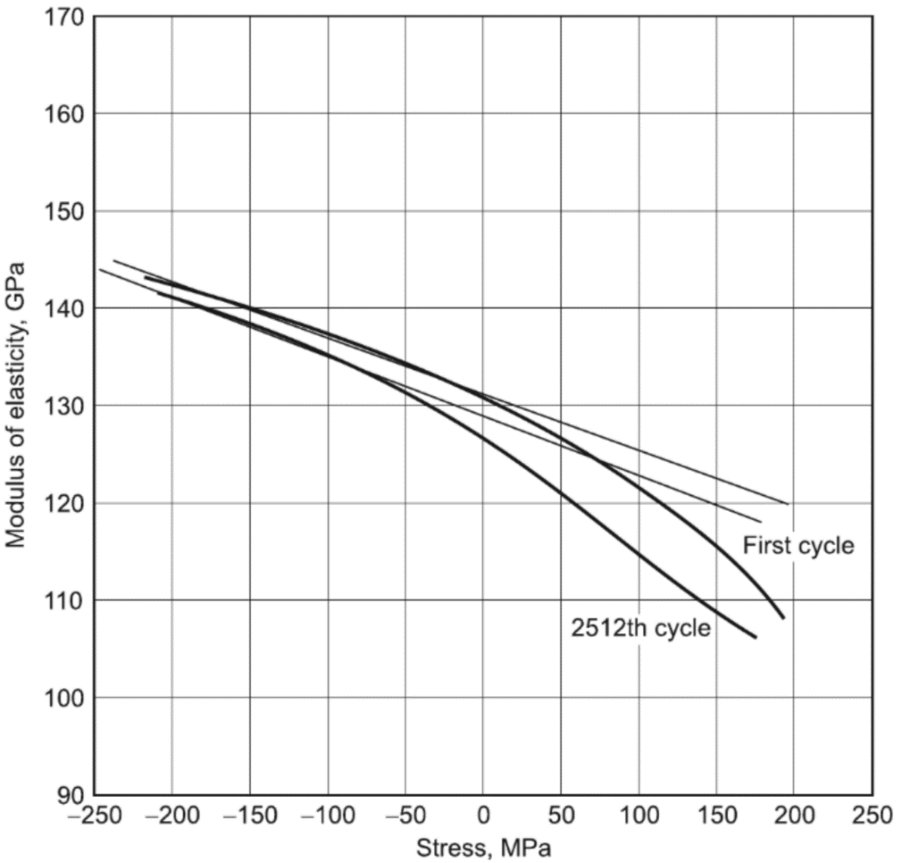




AS.014 4140 chromium-molybdenum alloy steel bar, monotonic and cyclic true stress-strain curves

Heat treatment: austenitized 999 °C (1830 °F), 1 h, oil quenched, tempered 399 °C (750 °F), 1 h, water quenched. Gage section size = 5.08 mm diam × 7.62 mm long (0.2 in. diam × 0.3 in. long). Strain rate = 0.5/min. Test condition: MT, monotonic tension; MC, monotonic compression; CT, cyclic tension; CC, cyclic compression. Composition: Fe-0.4C-1Cr-0.2Mo. UNS G41400

Source: P.N. Thielen, M.F. Fine, and R.A. Fournelle, Cyclic Stress Strain Relations and Strain-Controlled Fatigue of 4140 Steel, *Acta Metall.,* Vol 24 (No. 1), Jan 1976, p 1–10. As published in *Aerospace Structural Metals Handbook,* Vol 1, Code 1203, CINDAS/USAF CRDA Handbooks Operation, Purdue University, 1995, p 18



CI.058 Gray iron casting, modulus of elasticity-stress curves

Modulus of elasticity (E) for compression of first and 2512th cycle. At maximum compressive stress (0.0020 strain controlled) first cycle, E = 144.95 GPa; 2512th cycle, E = 144.20 GPa

Source: G.N.J. Gilbert, "The Cyclic Stress/Strain Properties and Fatigue Properties of a Flake Graphite Cast Iron Tested under Strain Control—A Detailed Study," Report 1621, British Cast Iron Research Association (BCIRA), 1985

Creep

Creep is a performance-based behaviour since it is not an intrinsic materials response.

It is defined as time-dependent deformation at absolute temperatures greater than one half the absolute melting

Diffusion-controlled mechanisms have significant effect on high temperature mechanical properties and performances.
For example, dislocation climb, concentration of vacancies, new slip systems, and grain boundary sliding all are diffusion-controlled and affect the behaviour of materials at high temperatures. In addition, corrosion or oxidation mechanisms, which are diffusion-rate dependent, will have an effect on the life time of materials at high temperatures.

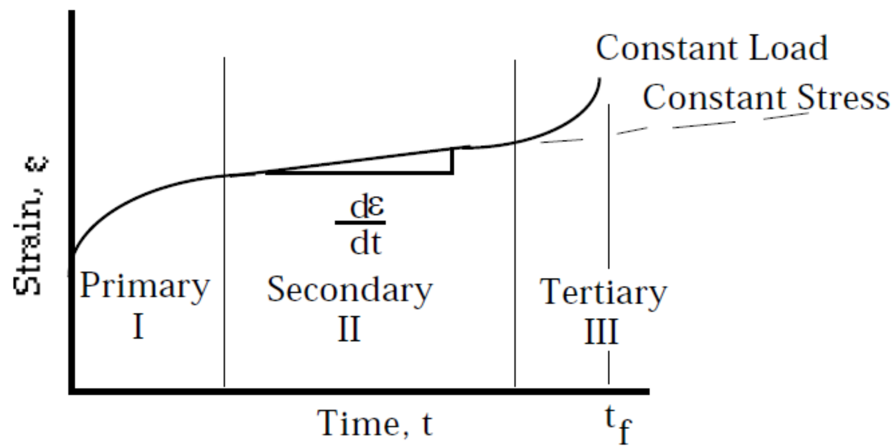


Figure 8.3 Strain time curve for a creep test

An empirical relation which describes the strain-time relation is $\varepsilon = \varepsilon_i + \varepsilon_t\big(1 - exp(rt)\big) + t\dot{\varepsilon_{ss}}$
where r is a constant, ε_t is the strain at the transition from primary to secondary creep and
ε_{ss} is the steady-state strain rate

$$\frac{d\varepsilon}{dt} = \frac{C\sigma^m}{d^b} e^{-\frac{Q}{kT}}$$ General creep equation

where E ε is the creep strain, *C* is a constant dependent on the material and the particular creep mechanism, *m* and *b* are exponents dependent on the creep mechanism, *Q* is the activation energy of the creep mechanism, σ is the applied stress, *d* is the grain size of the material, *k* is Boltzmann's constant, and *T* is the absolute temperature

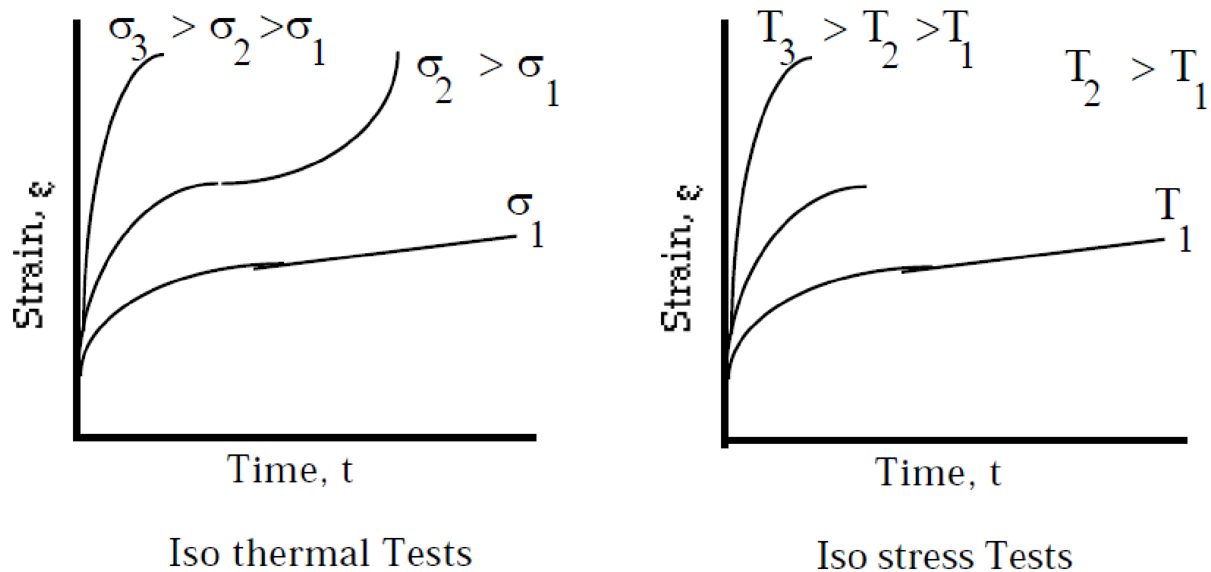


Figure 8.4 Effect of stress and temperature on strain time creep curves