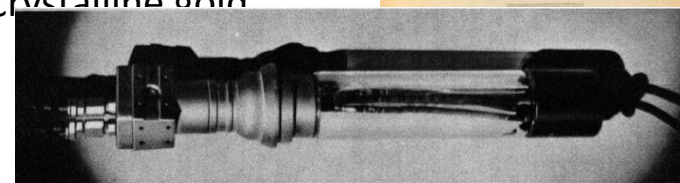
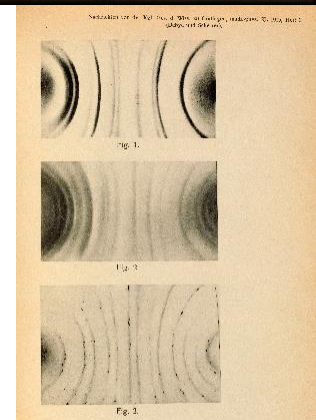
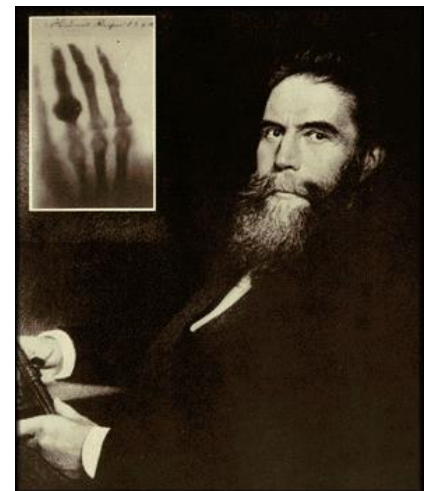


X-ray Diffraction

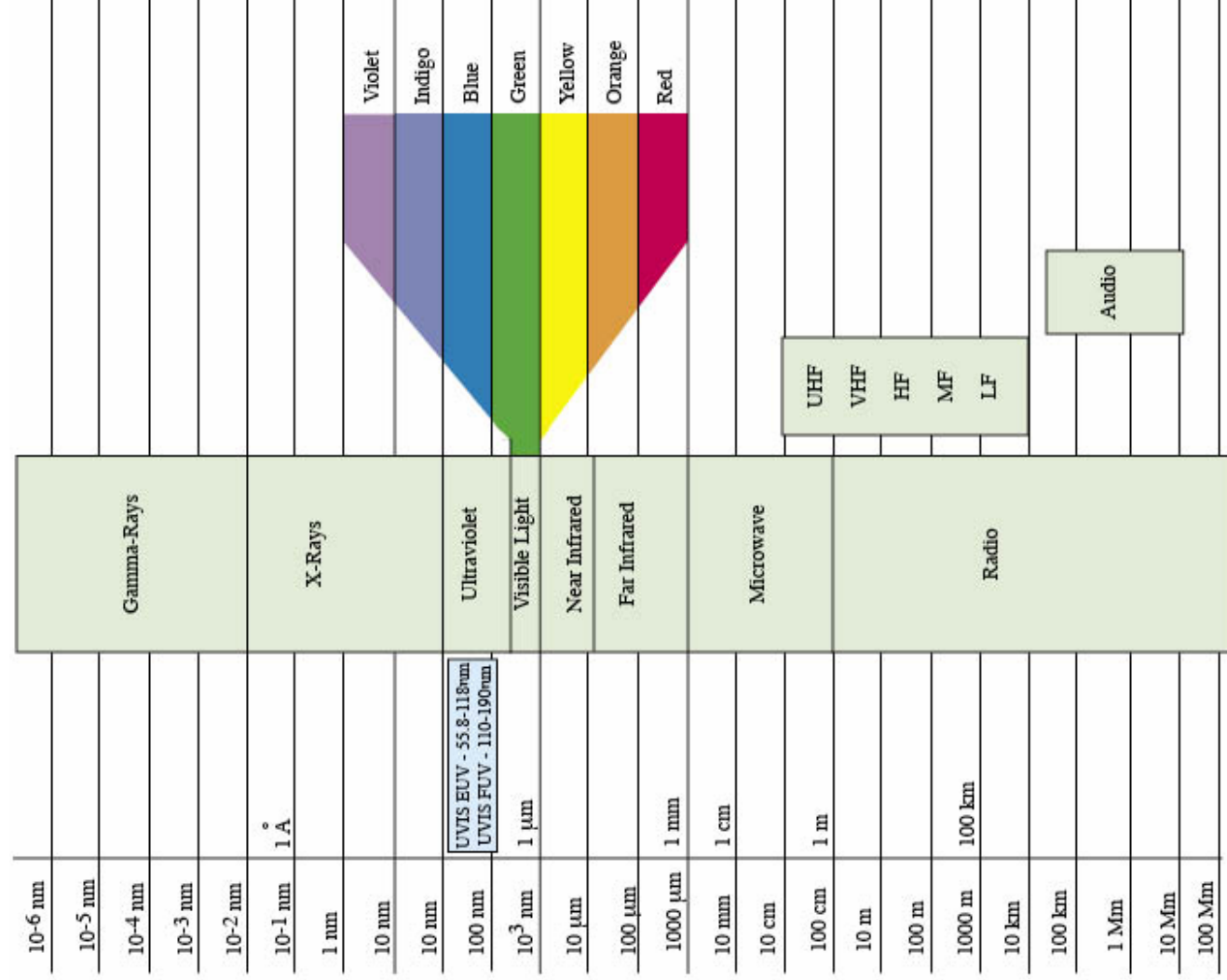
A Brief History of XRD

- 1895: Röntgen discovers X-Rays
 - received the first Nobel prize in physics in 1901
- 1912: Laue diffracts X-Rays from single crystal
 - 1914 Nobel prize in Physics
- 1912: Bragg analyzes crystal structures
 - 1915 Nobel prize in physics
- 1917: Ewald develops dynamical theory of X-Ray diffraction
- 1918: Scherrer uses X-Rays to determine crystallite size of nanocrystalline gold
- 1935: X-Ray powder diffractometer developed by Le Galley



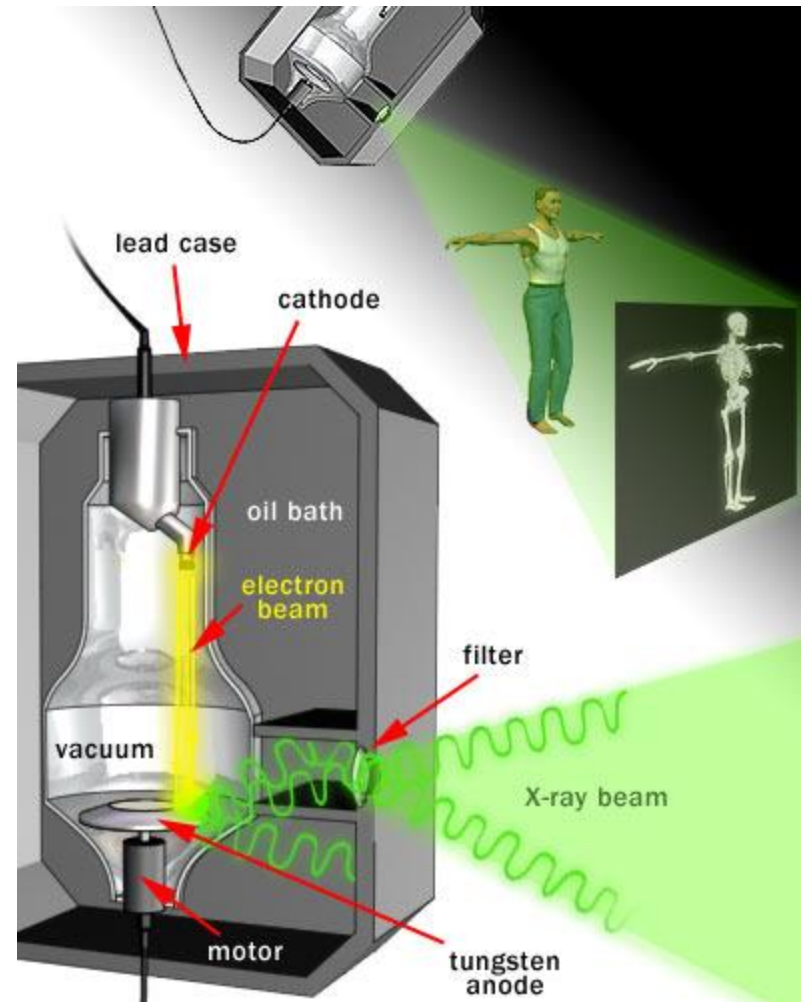
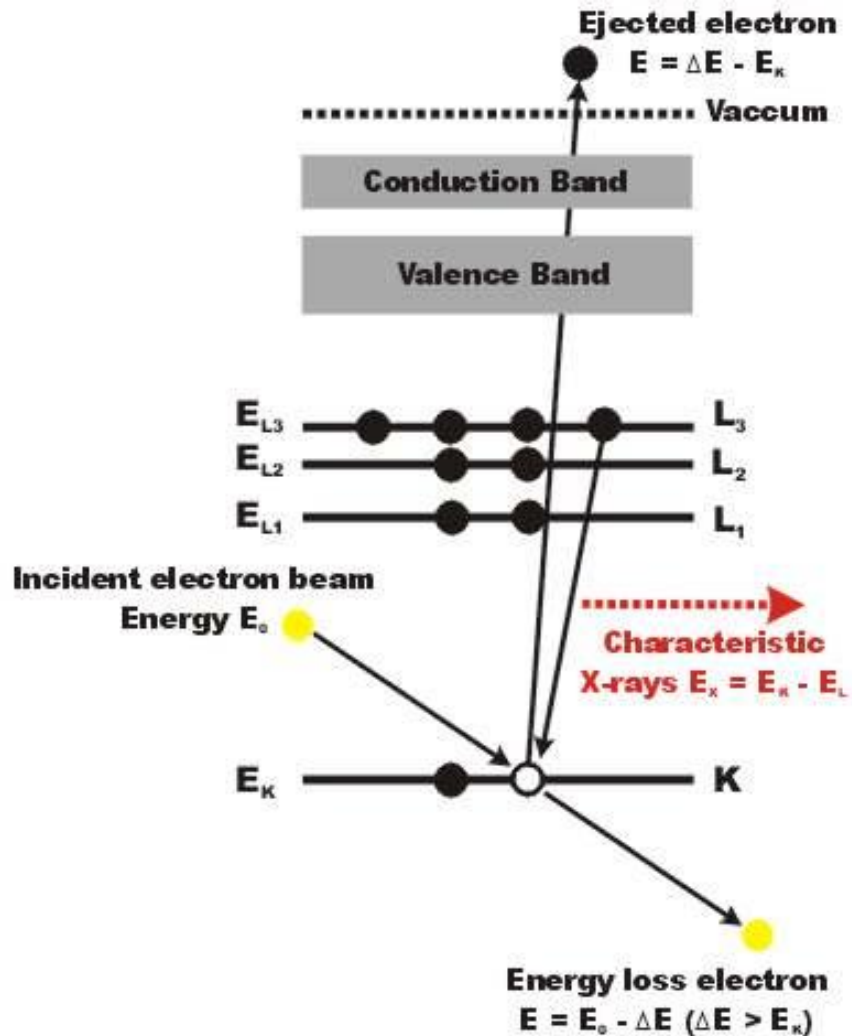
- 1947: First commercial powder diffractometer
- 1969: Establishment of the Joint Committee on Powder Diffraction Standards (JCPDS), now International Center for Diffraction Data (ICDD)

Electromagnetic Spectrum

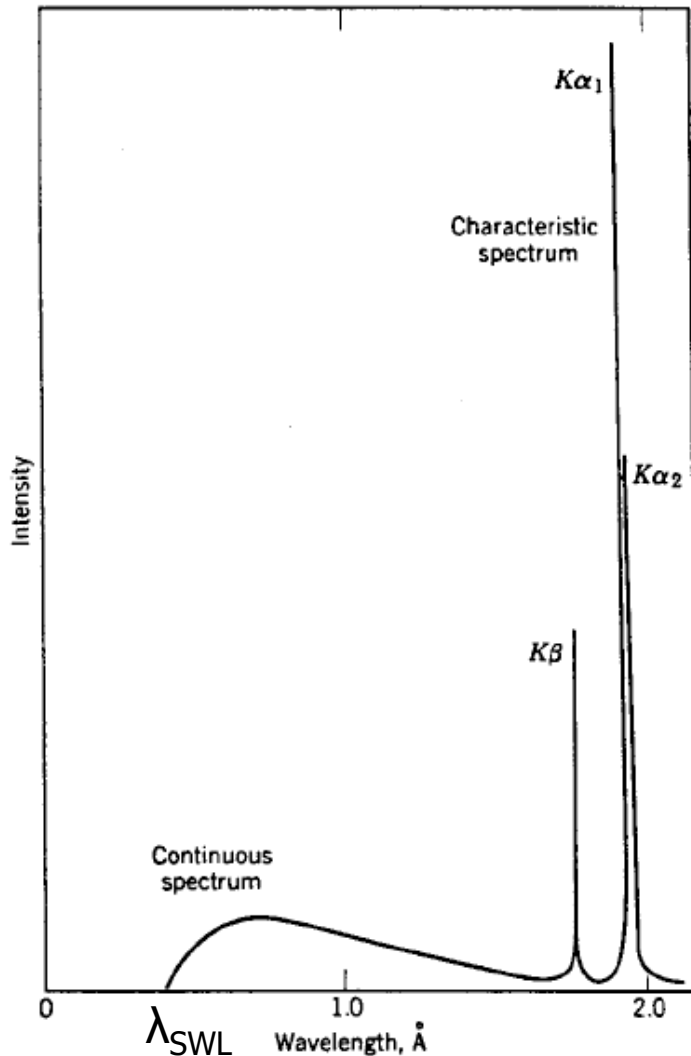


nm=nanometer, \AA =angstrom, μm =micrometer, mm=millimeter,
cm=centimeter, m=meter, km=kilometer, Mm=Megameter

Generation of X-rays



X-ray Spectrum from an Iron target



- Short Wavelength Limit $\lambda_{SWL} \text{ (nm)} = \frac{12400}{V}$
- Continuous spectrum $I_{CS} = AiZV^m$
- Characteristic X-ray Moseley's Law

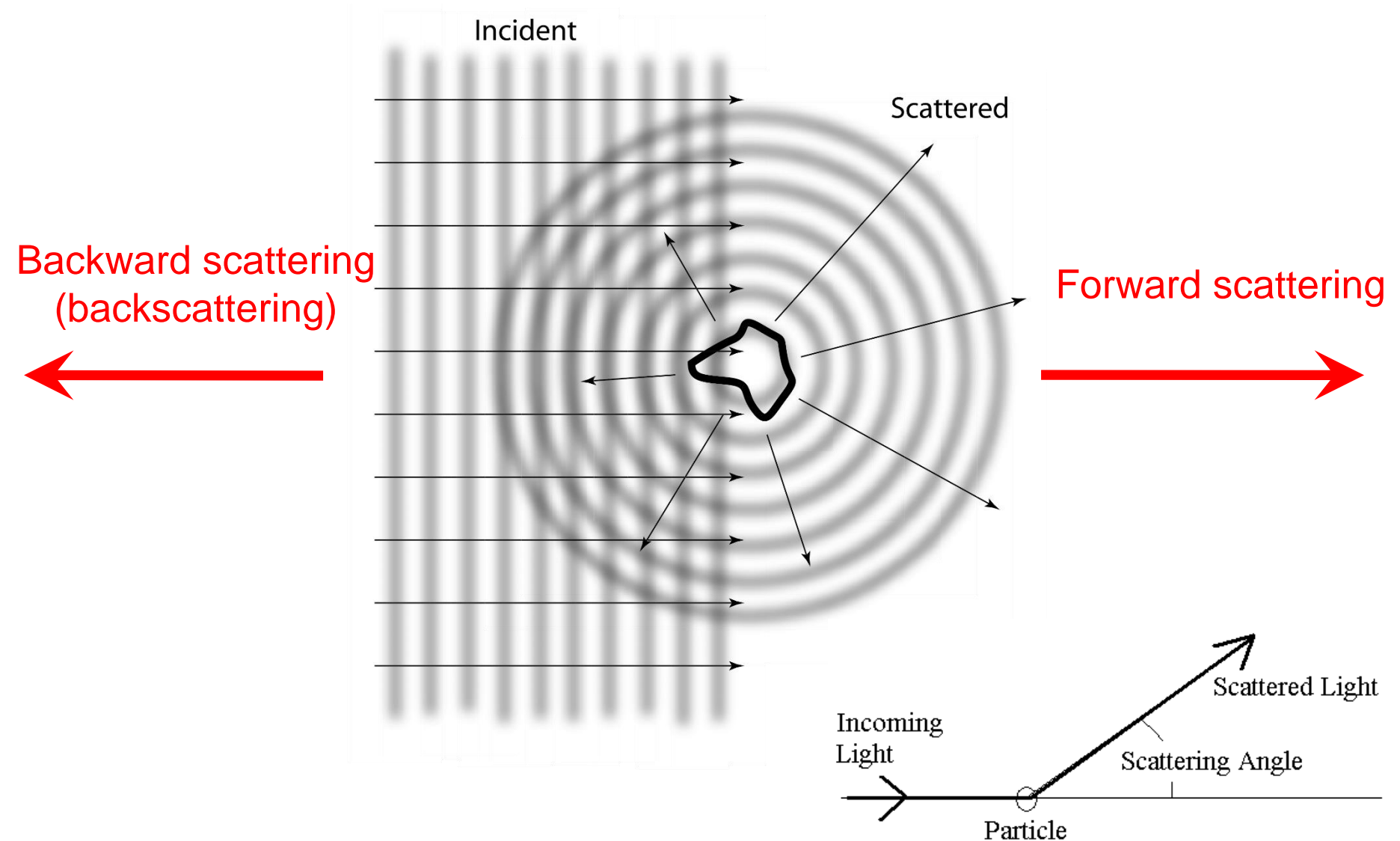
$$\sqrt{\nu} = C(Z - \sigma)$$

$$I_K = Bi(V - V_k)^n$$

Scattering fundamentals

- *Scattering* can be broadly defined as the *redirection of radiation out of the original direction of propagation*, usually due to interactions with molecules and particles
- Reflection, refraction, diffraction etc. are actually all just forms of scattering
- Matter is composed of discrete electrical charges (atoms and molecules – dipoles)
- Light is an oscillating EM field – excites charges, which radiate EM waves
- These radiated EM waves are *scattered waves*, excited by a source external to the scatterer
- The *superposition of incident and scattered EM waves* is what is observed

Scattering geometry

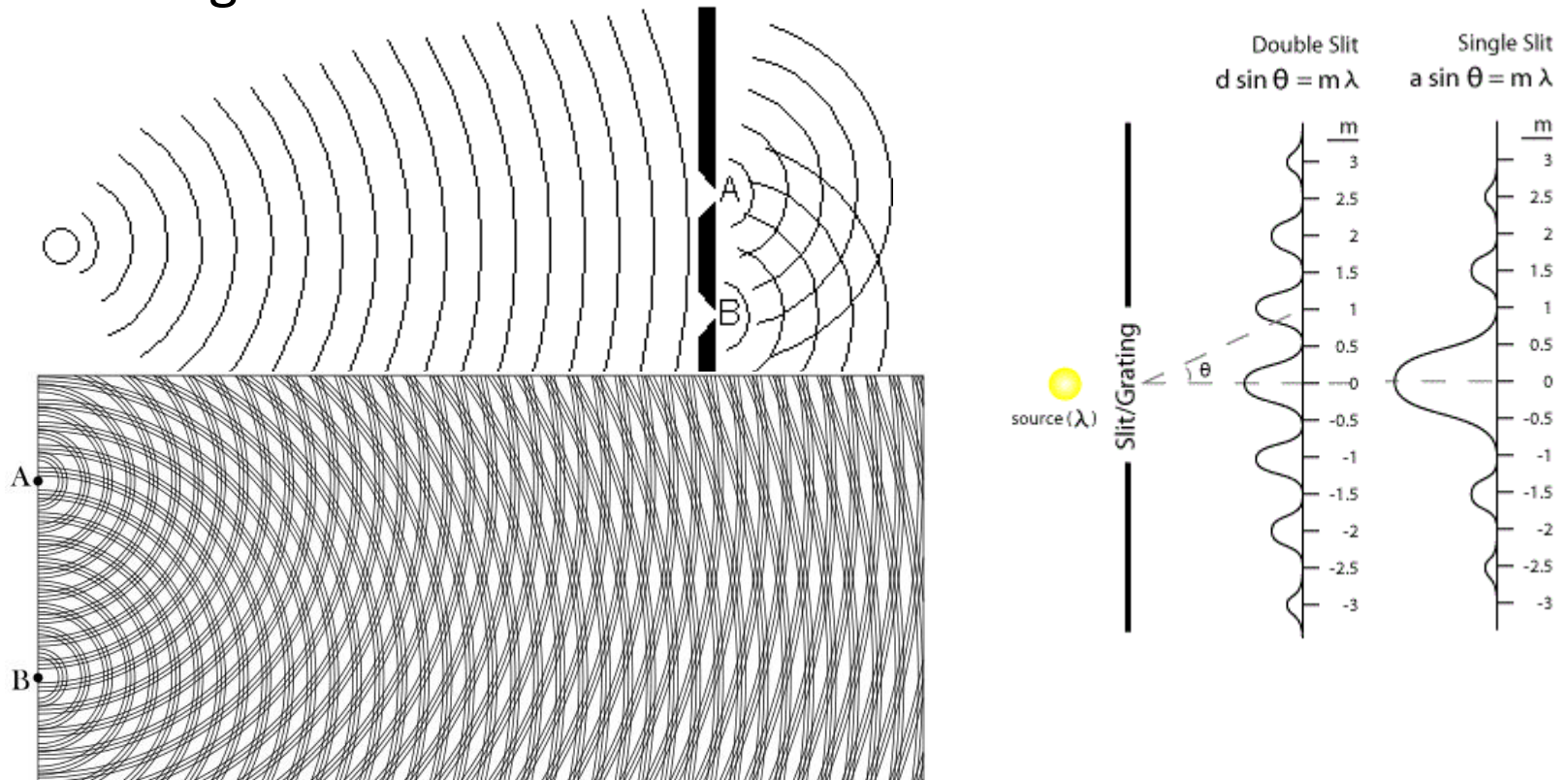


Types of scattering

- *Elastic scattering* – the wavelength (frequency) of the scattered light is the same as the incident light (*Rayleigh and Mie scattering*)
- *Inelastic scattering* – the emitted radiation has a wavelength different from that of the incident radiation (*Raman scattering, fluorescence*)
- *Quasi-elastic scattering* – the wavelength (frequency) of the scattered light shifts (e.g., in moving matter due to Doppler effects)

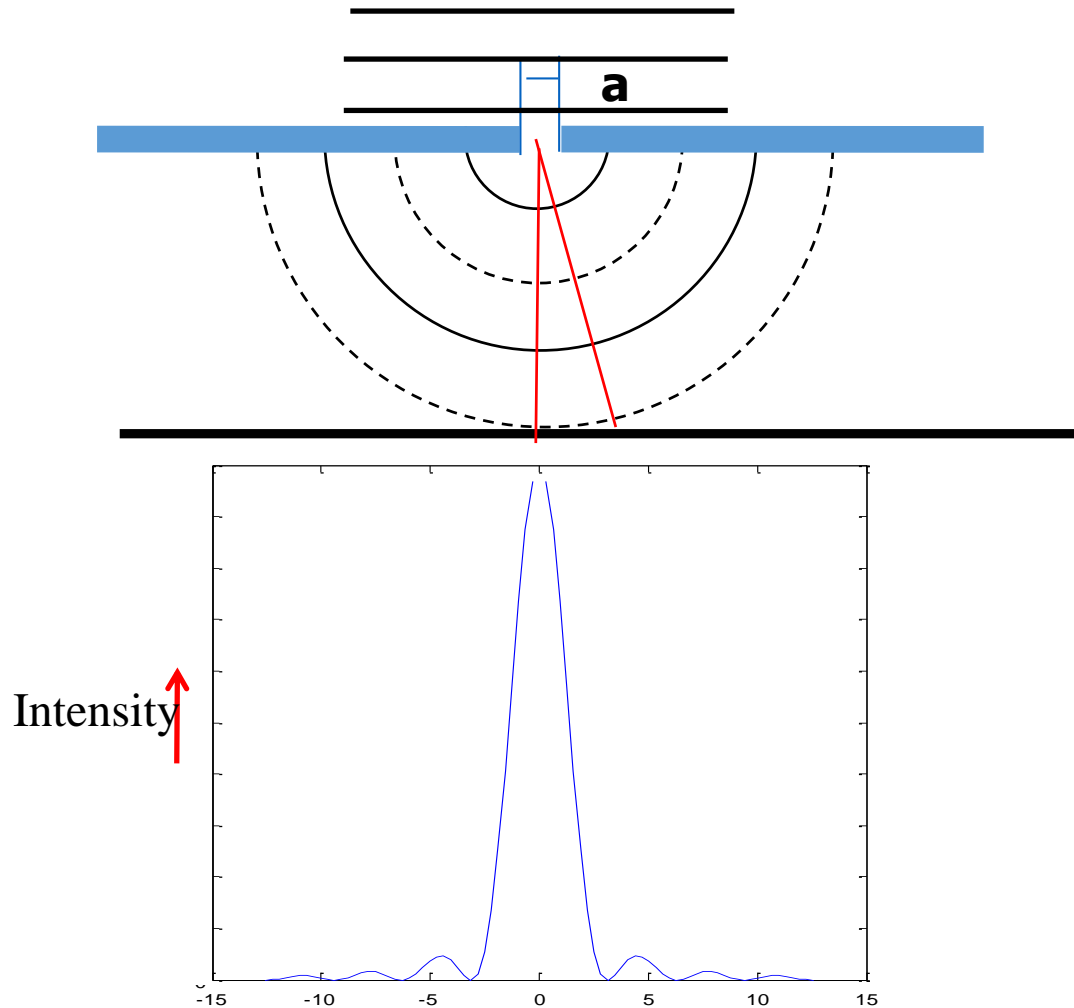
How Diffraction occurs

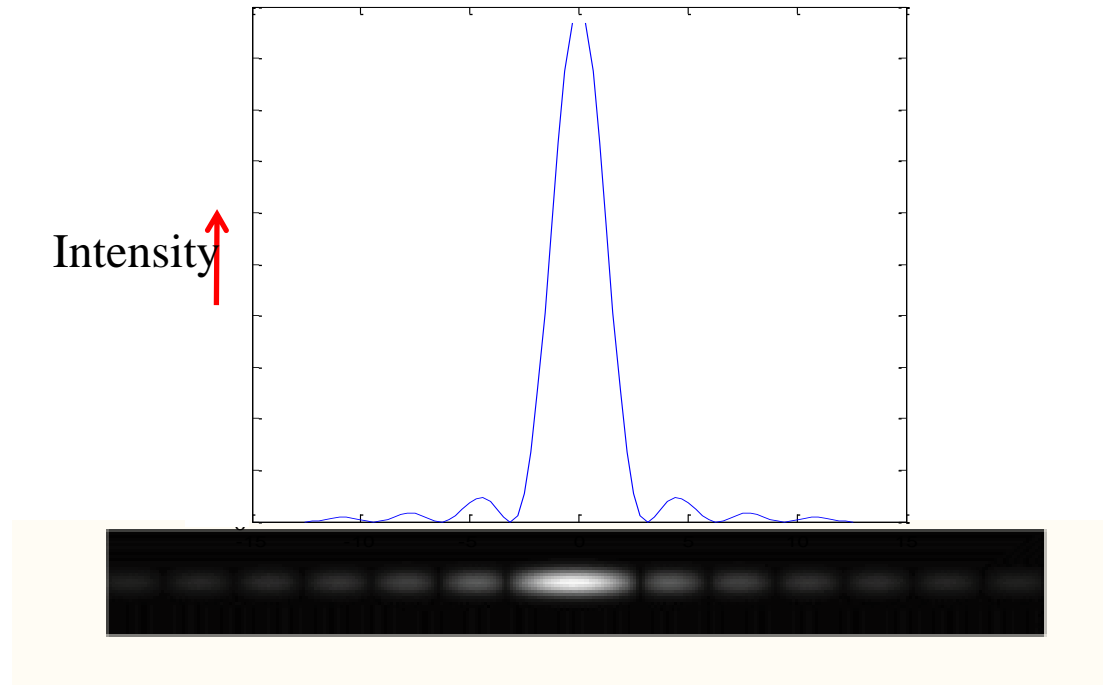
- Diffraction occurs when objects in a periodic array scatter radiation coherently, producing constructive interference at specific angles



- X-Rays can diffract from a periodic array of elastic scatterers, such as atoms in a crystal

Diffraction of light through an aperture





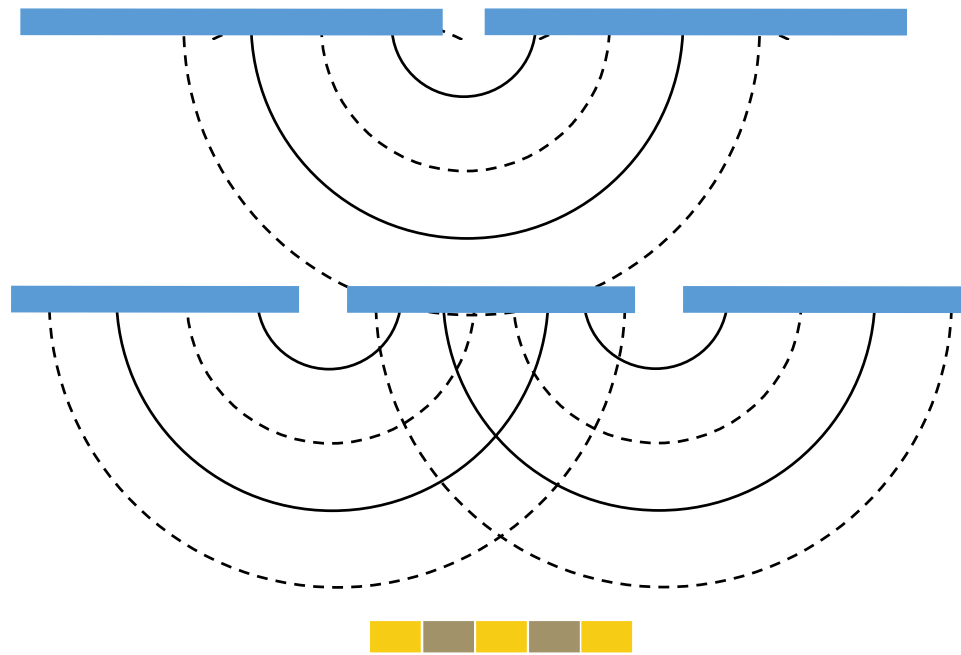
Minima

Maxima

$$\frac{\sin \theta}{\lambda} = \frac{n}{a} \quad n = 0, 1, \dots$$

$$\frac{\sin \theta}{\lambda} = \frac{2n+1}{2a} \quad n = 1, 2, \dots$$

Young's Double slit experiment



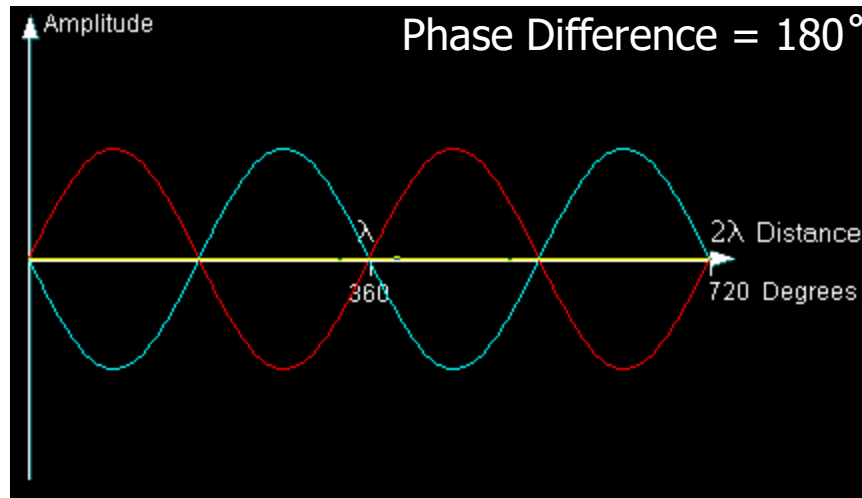
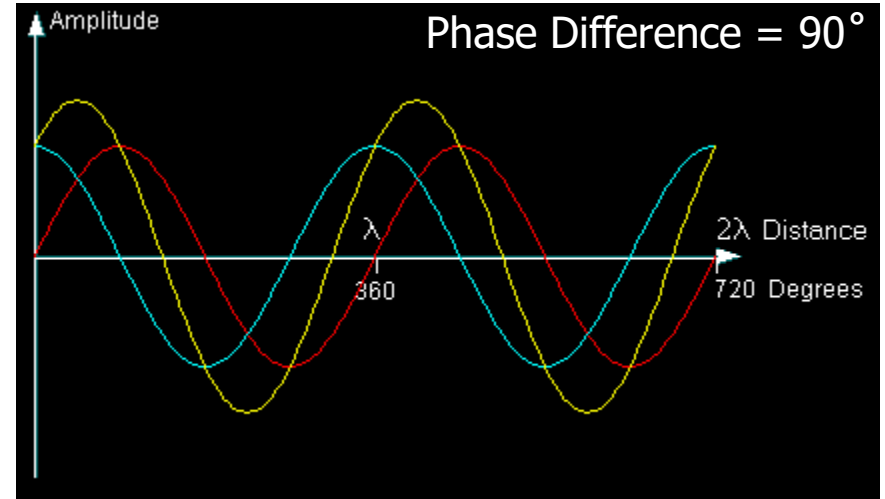
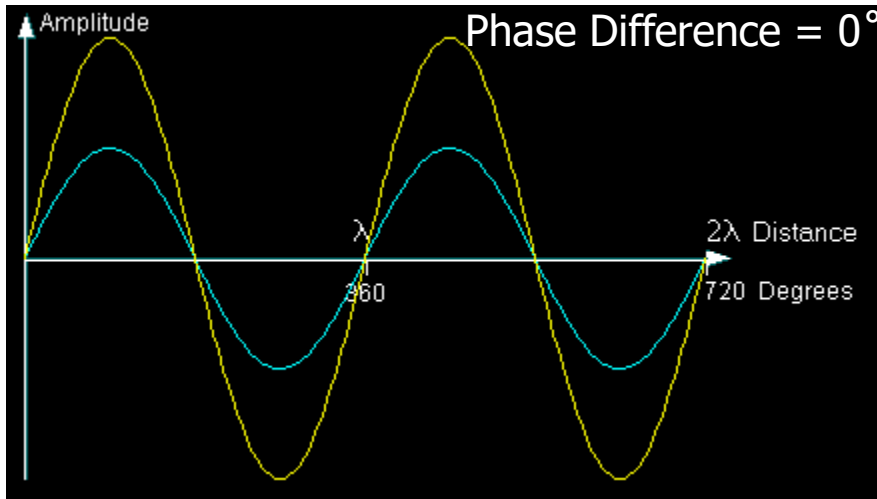
$$d \sin\theta = m\lambda, m = 1, 2, 3, \dots$$

Constructive Interference

$$d \sin\theta = (m + \frac{1}{2})\lambda, m = 1, 2, 3, \dots$$

Destructive Interference

Interference



Elastic Scattering

- When x-rays or electrons interact with matter, the dominant effect is *scattering*.
- Considering x-rays and electrons as waves we deal with elastic scattering (rather than as particles, where we deal with inelastic scattering)
- For elastic scattering, x-rays and electrons are scattered with no loss of energy, and give rise to scattered radiation of the same wavelength

Constructive Interference

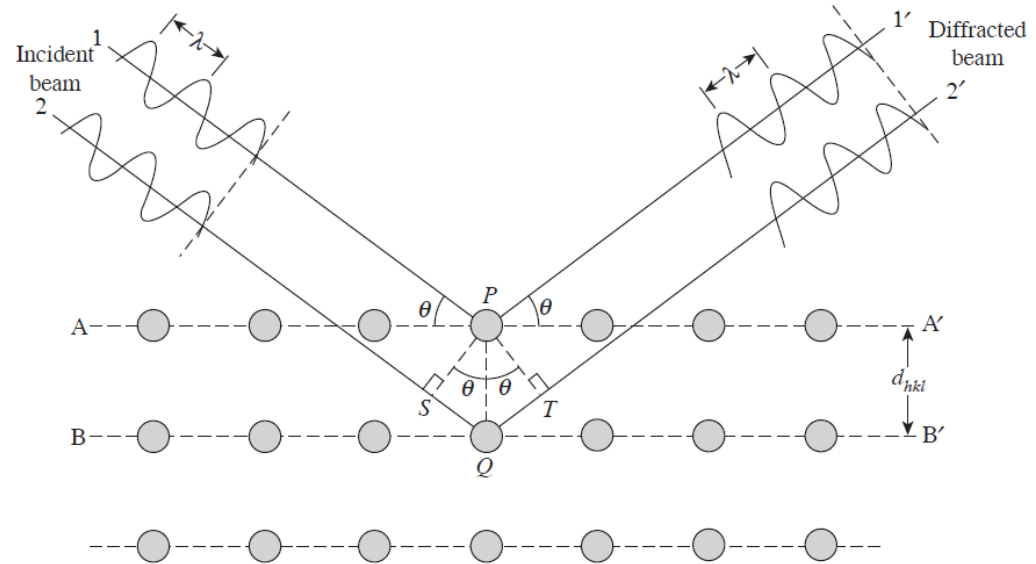
- The distance between atoms (d_{hkl}) are on the same order of size as the wavelength of an x-ray (Cu Ka = 1.54 Å)
- Interference phenomena is concentrated in directions related to the crystal lattice
- The intensity of the diffracted x-rays gives rise to peaks for each set of wave vectors which make up diffraction patterns
- The positions of the atoms in the material (the crystal lattice of the solid) and the wavelength of the x-rays determine the positions and intensities of the diffracted peaks.

Another kind of scattering, incoherent (Compton), is easiest understood in terms of the particle nature of photons: the photon deviates from path and electron takes part of its energy. The scattered photon has lost energy (so has a longer wavelength), and there is no relationship between the phases of the two waves. There is no interference and of little significance here (though it is for XRF) and we will not consider it further.

Conditions Required for X-Ray Diffraction

Bragg's law - a model to describe the position of observed diffraction peaks

$$\lambda = 2d_{hkl} \sin \theta$$



- Constructive interference only occurs when Bragg's law is satisfied for parallel planes of atoms, with a space d_{hkl} between them
- Each plane of atoms produces a diffraction peak at a specific angle θ
- Wavelengths of the excited filament material should be similar in dimension with the interplanar spacing d (most common emission being copper K_{α} 1.5418 Å)

Conditions Required for X-Ray Diffraction

Miller indices (hkl) define a series of parallel planes in a crystal with interplanar spacing d

The interplanar distance is dependent on the Miller indices. When combined with Bragg's law:

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \qquad \sin^2 \theta = \frac{\lambda^2}{4a^2} (h^2 + k^2 + l^2)$$

This equation helps us determine the Miller indices of crystal planes that diffract x-rays

For example: {001} is the planar index for $(h^2+k^2+l^2)=1$

Or {110} is the planar index for $(h^2+k^2+l^2)=2$

This calculation is not practical because Miller indices for the diffracting planes and the diffraction angles for a known wavelength are presented for all crystalline materials by the International Center for Diffraction Data (ICDD)

Diffraction Data is Essential

PDF # 860740, Wavelength = 1.54060 (Å)

86-0740 Quality: C

CAS Number:

Molecular Weight: 502.32

Volume[CD]: 521.26

Dx: 3.200 Dm:

Sys: Hexagonal

Lattice: Primitive

S.G.: P6₃/m (176)

Cell Parameters:

a 9.352 b c 6.882

α β γ

I/I_{cor}: 1.04

Rad: CuK α 1

Lambda: 1.54060

Filter:

d-sp: calculated

ICSD #: 081442

Mineral Name:

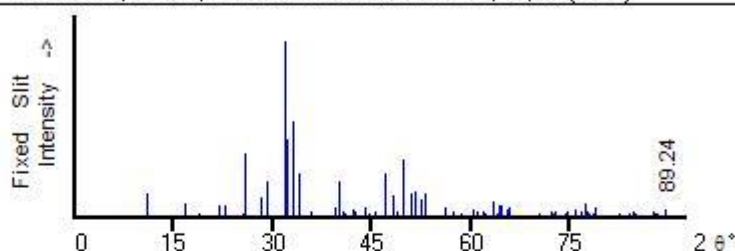
Hydroxylapatite

Ca₅(P₃O₄)₃(OH)

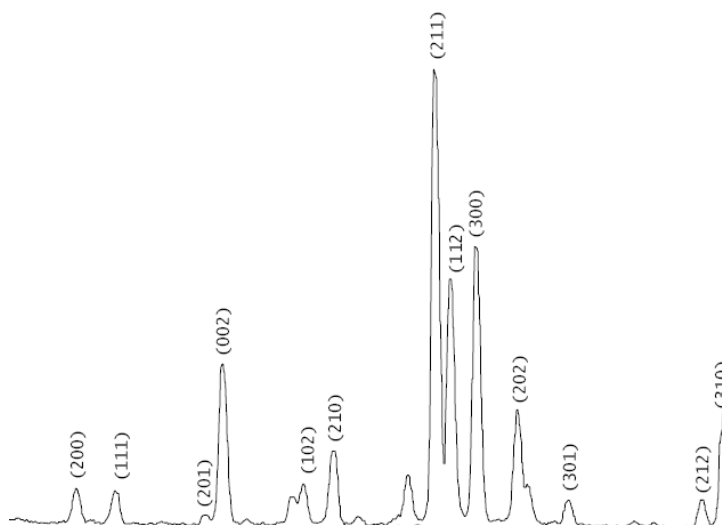
Calcium Phosphate Hydroxide

Ref: Calculated from ICSD using POWD-12++, (1997)

Ref: Tomita, K et al., J. Mineral. Petrol. Econ. Geol., 91, 11 (1996)



2 θ	Int-f	h	k	l	2 θ	Int-f	h	k	l	2 θ	Int-f	h	k	l
10.915	125	1	0	0	45.402	32	2	0	3	63.950	17	5	1	0
16.892	69	1	0	1	46.947	248	2	2	2	64.134	64	3	0	4
18.964	22	1	1	0	48.350	116	1	3	2	64.485	58	3	2	3
21.931	66	2	0	0	48.985	33	2	3	0	65.537	44	5	1	1
22.976	64	1	1	1	49.620	323	2	1	3	65.714	46	3	3	2
25.501	14	2	0	1	50.861	134	2	3	1	66.761	7	4	1	3
25.872	361	0	0	2	51.679	139	1	4	0	66.846	8	4	2	2
28.154	112	1	0	2	52.388	102	4	0	2	67.543	1	2	2	4
29.149	198	2	1	0	53.195	135	0	0	4	68.660	4	1	3	4
31.973	999*	2	1	1	54.479	4	1	0	4	69.176	1	1	0	5
32.275	437	1	1	2	56.218	54	3	2	2	69.593	2	6	0	0
33.157	542	3	0	0	56.790	2	5	0	0	70.188	16	1	5	2
34.167	245	2	0	2	56.988	6	1	1	4	70.694	2	4	3	0
35.697	31	3	0	1	57.364	25	3	1	3	71.204	2	5	0	3
38.473	2	2	2	0	58.215	1	2	0	4	71.378	2	1	1	5
39.364	51	2	1	2	58.490	12	5	0	1	71.963	5	4	0	4
40.110	203	1	3	0	58.678	8	4	1	2	72.209	29	4	3	1
40.726	30	2	2	1	59.234	3	3	3	0	72.469	15	2	0	5
40.852	19	1	0	3	60.434	43	2	4	0	72.877	27	5	2	0
42.290	41	1	3	1	60.891	30	3	3	1	73.380	3	3	3	3
42.528	29	3	0	2	61.806	33	1	2	4	74.373	17	5	2	1
43.928	52	1	1	3	62.071	12	2	4	1	74.459	23	2	4	3
44.722	13	4	0	0	63.418	83	5	0	2	75.205	7	2	3	4



Conditions Required for X-Ray Diffraction

Not all diffracted lights from crystal planes that satisfy Bragg's law are detectable. Detection of the diffracted light depends on its intensity

X-ray diffraction in a crystal happens in all direction from all atoms within the crystal that satisfy the Bragg's law

All these atoms contribute to the intensity of the diffracted light

An X-ray can be scattered by an electron in all directions in space

The intensity of the scattered light is a function of the angle (2θ) between the incident photon and the scattered:

$$I(2\theta) = \frac{I_o}{r^2} K \frac{1 + \cos^2(2\theta)}{2}$$

- I_o is the intensity of the incident photon
- r is the distance between the electron and the detector
- K is a constant depending on the properties of the atom
- The mathematical term represents the effect of the scattering angle on intensity – Polarization factor

Conditions Required for X-Ray Diffraction

Total intensity of the scattered (diffracted) X-rays from an atom at a specific scattering angle is less than the theoretical intensity of X-rays generated by the electrons

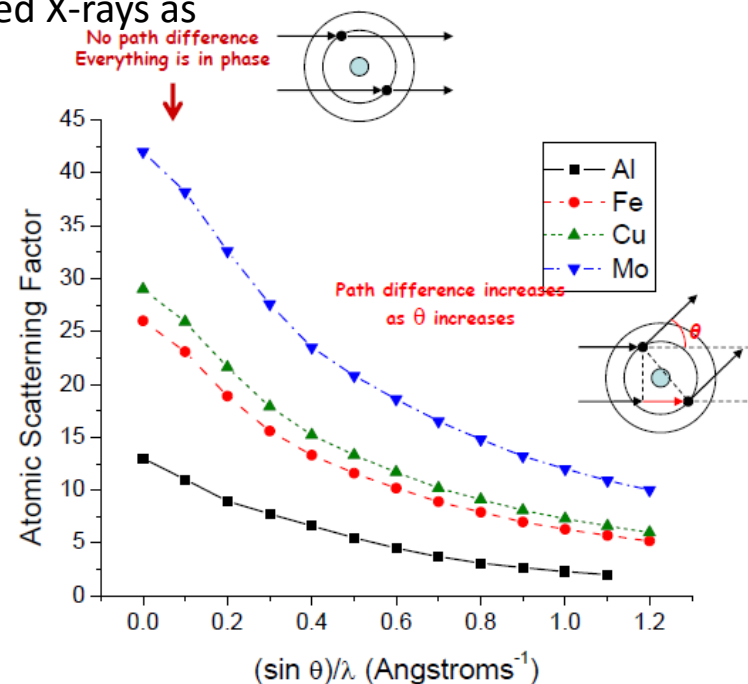
The reason is absorption (fluorescence) of some of the diffracted X-rays by electrons at suitable positions around the atom

Scattered X-rays follow different paths with different path lengths for each diffraction angle

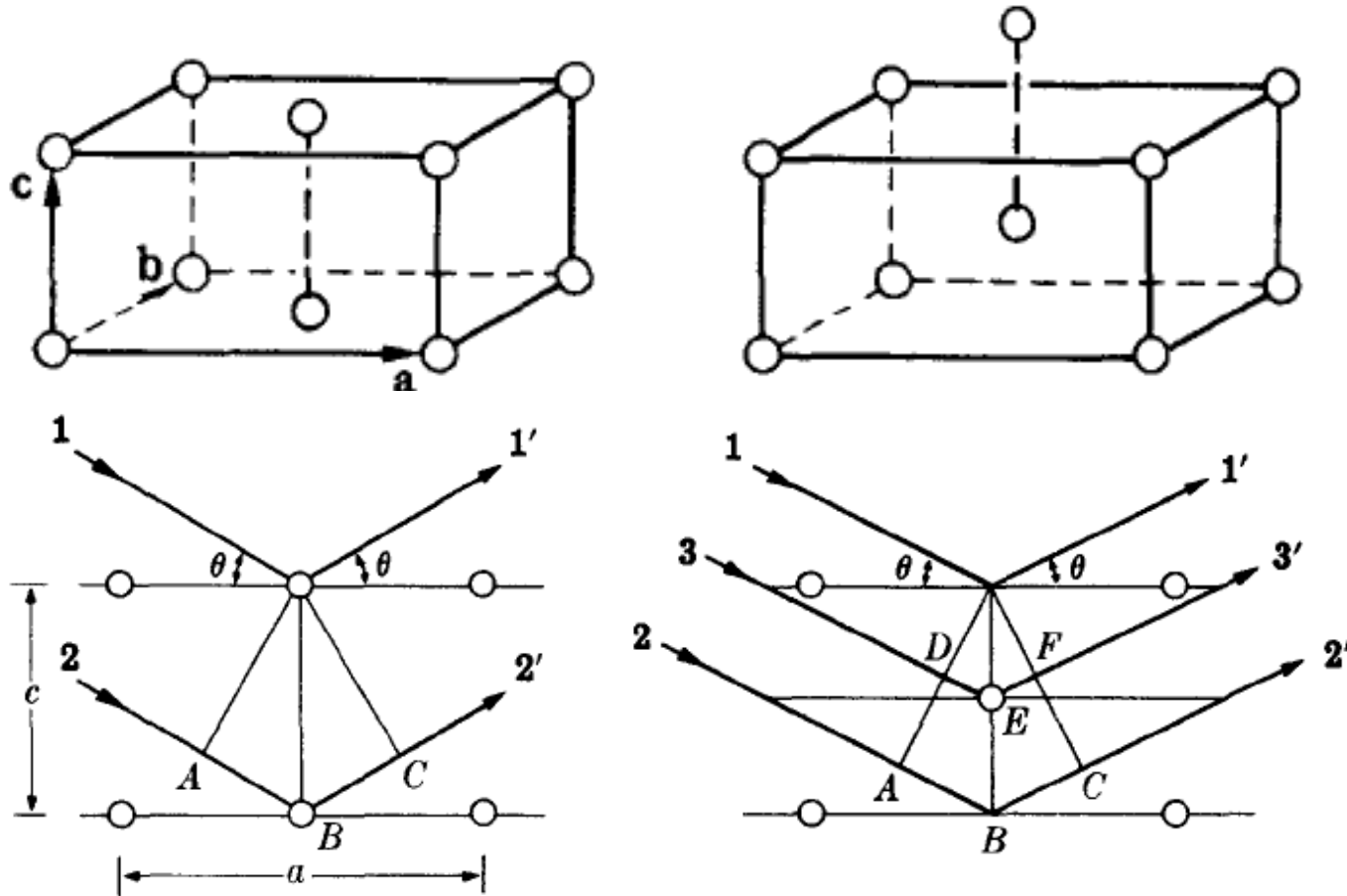
Atomic scattering factor determines the intensity of the diffracted X-rays as

$f = \text{Number of X-rays scattered from an atom/}$

$\text{Number of X-rays scattered from all electrons in the atom}$



Structural photon absorption



(a) Base-centered and (b) body-centered orthorhombic unit cells.

Conditions Required for X-Ray Diffraction

Structural absorption of diffracted X-ray intensity is determined from the Structural factor (F)

$$F_{hkl} = \sum_n^N f_n \exp [2\pi i(hu_n + kv_n + lw_n)]$$

It is independent of the shape (cubic, hexagonal, etc) or size of the crystal structure

In a unit cell within the (hkl) plane and consisting of N atoms, the position of the nth atom is defined as u_n, v_n, w_n

Structural factor can be calculated if the atomic scattering factor f_n is known

Conditions Required for X-Ray Diffraction

Structural absorption of diffracted X-ray intensity is determined from the Structural factor (F)

$$F_{hkl} = \sum_n^N f_n \exp [2\pi i(hu_n + kv_n + lw_n)]$$

For a plane within a body centered cubic crystal, the structural factor in the (001) and (002) planes are found as:

$$\begin{aligned} F_{001} &= f \exp [2\pi i(0 + 0 + 1 \times 0)] + f \exp [2\pi i(0 \times 0.5 + 0 \times 0.5 + 1 \times 0.5)] \\ &= f + f \exp [\pi i] = 0 \end{aligned}$$

$$\begin{aligned} F_{002} &= f \exp [2\pi i(0 + 0 + 2 \times 0)] + f \exp [2\pi i(0 \times 0.5 + 0 \times 0.5 + 2 \times 0.5)] \\ &= f + f \exp [2\pi i] = 2f \end{aligned}$$

Simple crystal structures like BCC, FCC, SC have the same structural factors in general

Structure Factor

$$F_{hkl} = \sum_1^N f_n e^{2\pi i(hu_n + kv_n + lw_n)}$$

Intensity of the diffracted beam $\propto |F|^2$

- h, k, l : indices of the diffraction plane under consideration
- u, v, w : co-ordinates of the atoms in the lattice
- N : number of atoms
- f_n : scattering factor of a particular type of atom

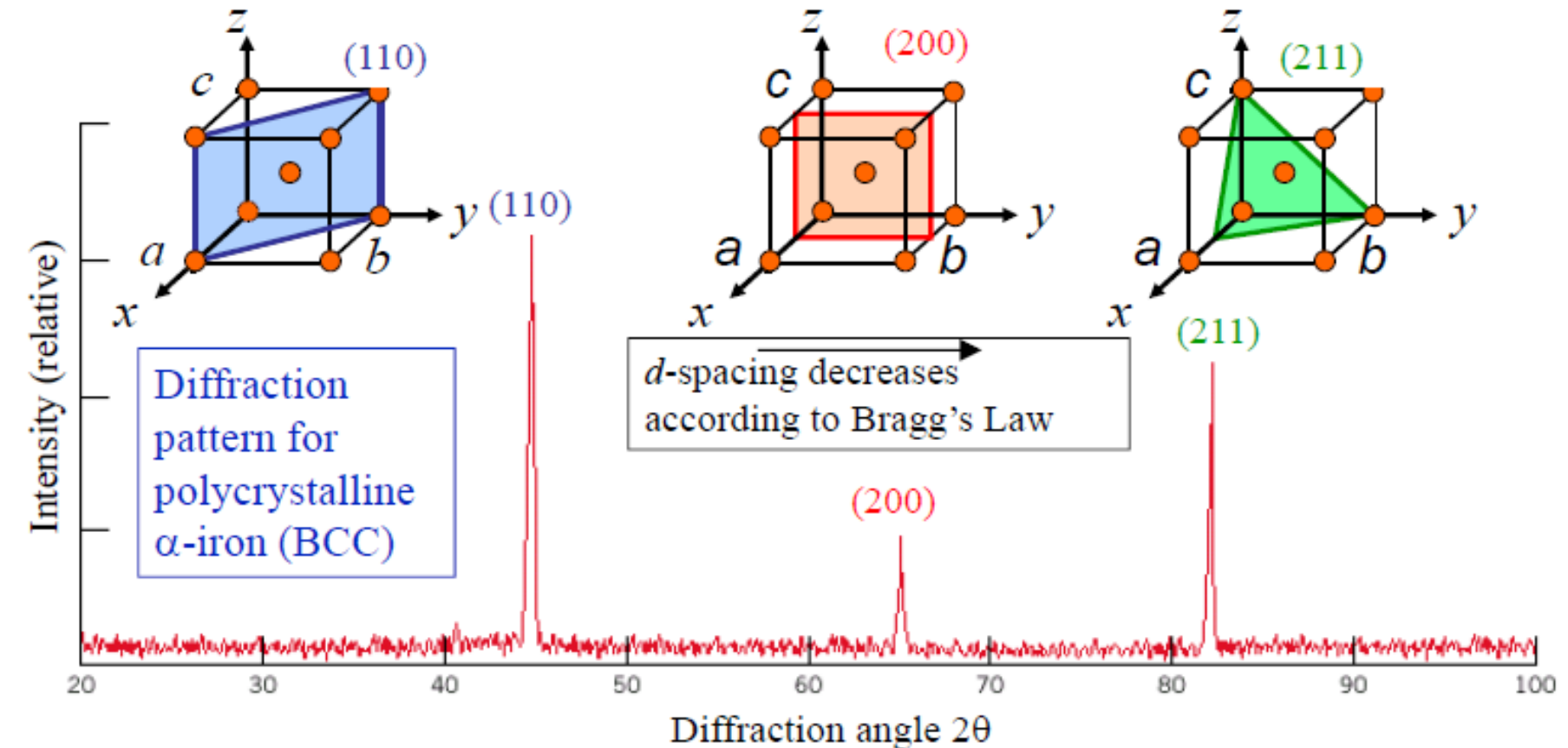
Bravais Lattice	Reflections possibly present	Reflections absent
Simple	All	None
Body Centered	$(h+k+l)$: Even	$(h+k+l)$: Odd
Face Centered	$h, k,$ and l unmixed i.e. all odd or all even	$h, k,$ and l : mixed

Systematic Absences

Permitted Reflections

Simple Cubic	$(100), (110), (111), (200), (210), (211),$ $(220), (300), (221) \dots\dots\dots$
BCC	$(110), (200), (211), (220), (310), (222) \dots\dots$
FCC	$(111), (200), (220), (311) \dots\dots$

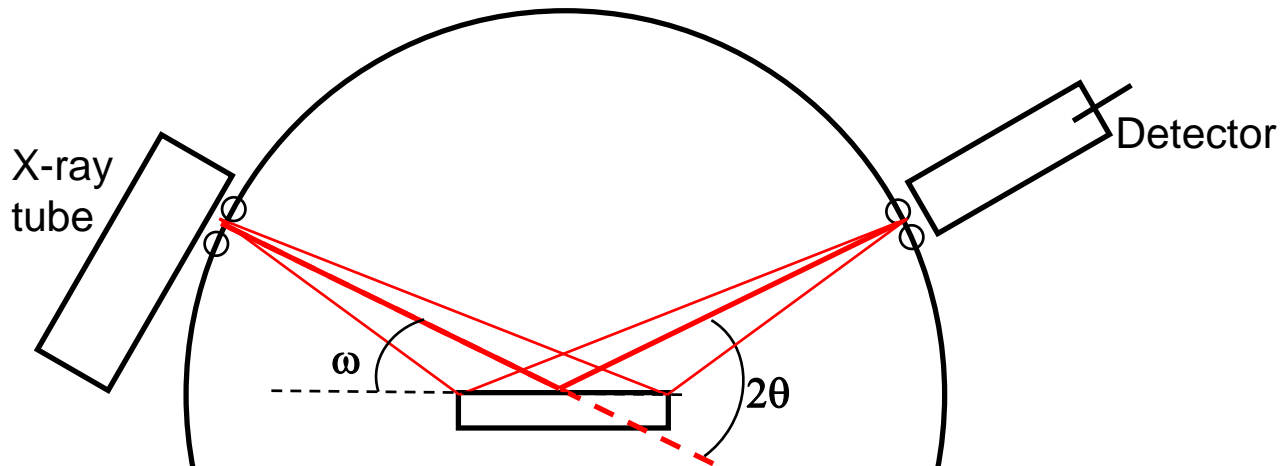
The result of structural absorption



→ $h+k+l$ even are only allowed according to previously shown selection rules for BCC.

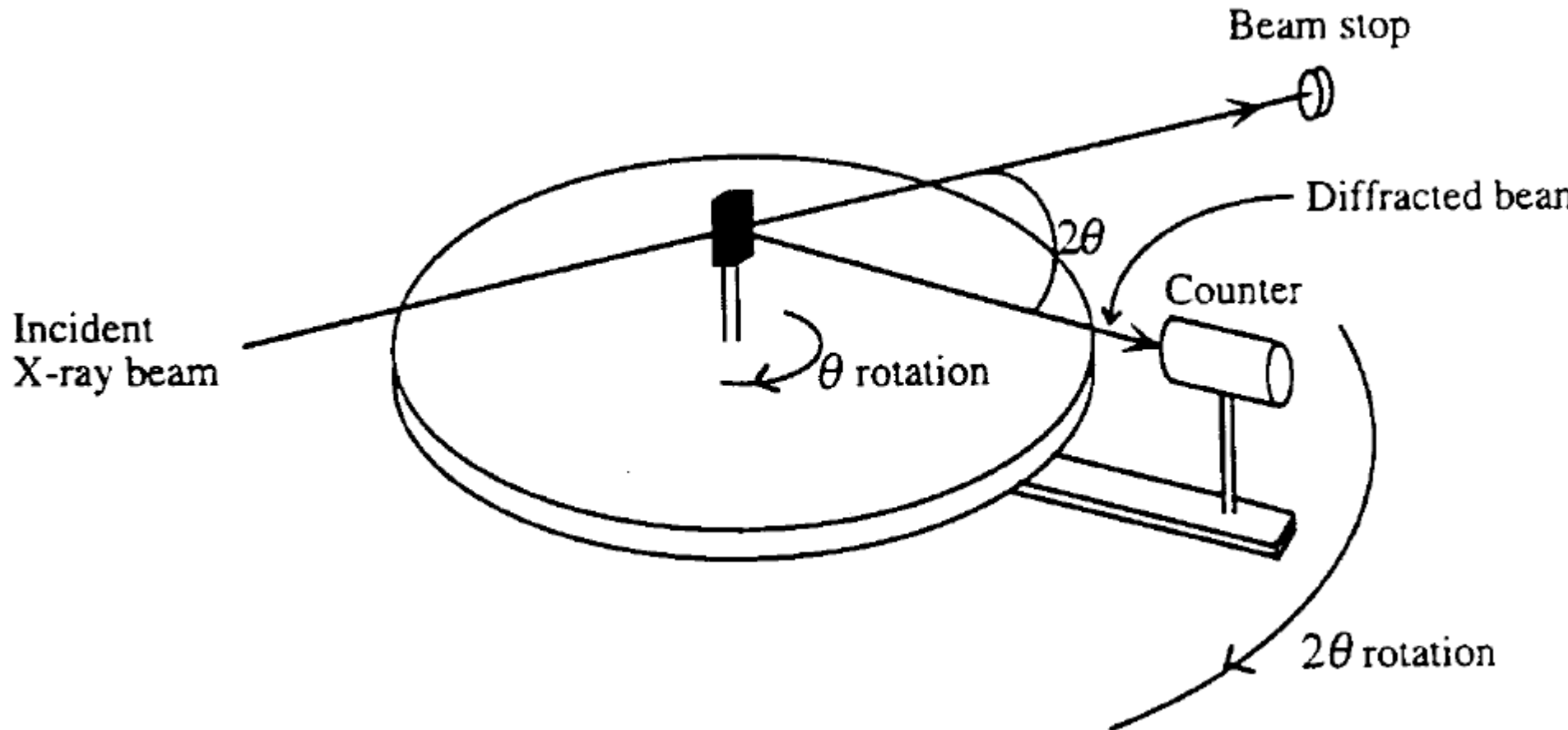
Diffraction Instrument

- Modern powder diffractometers typically use the Bragg-Brentano parafocusing geometry
- The incident angle between the X-Ray source and the sample is ω
The diffraction angle, between the incident beam and the detector angle, is 2θ
- The Bragg-Brentano geometry constrains ω to be always $\frac{1}{2}$ of the detector angle 2θ
- Two main diffractometer models: The tube is fixed, sample and detector rotate – The sample is fixed, tube and detector rotate



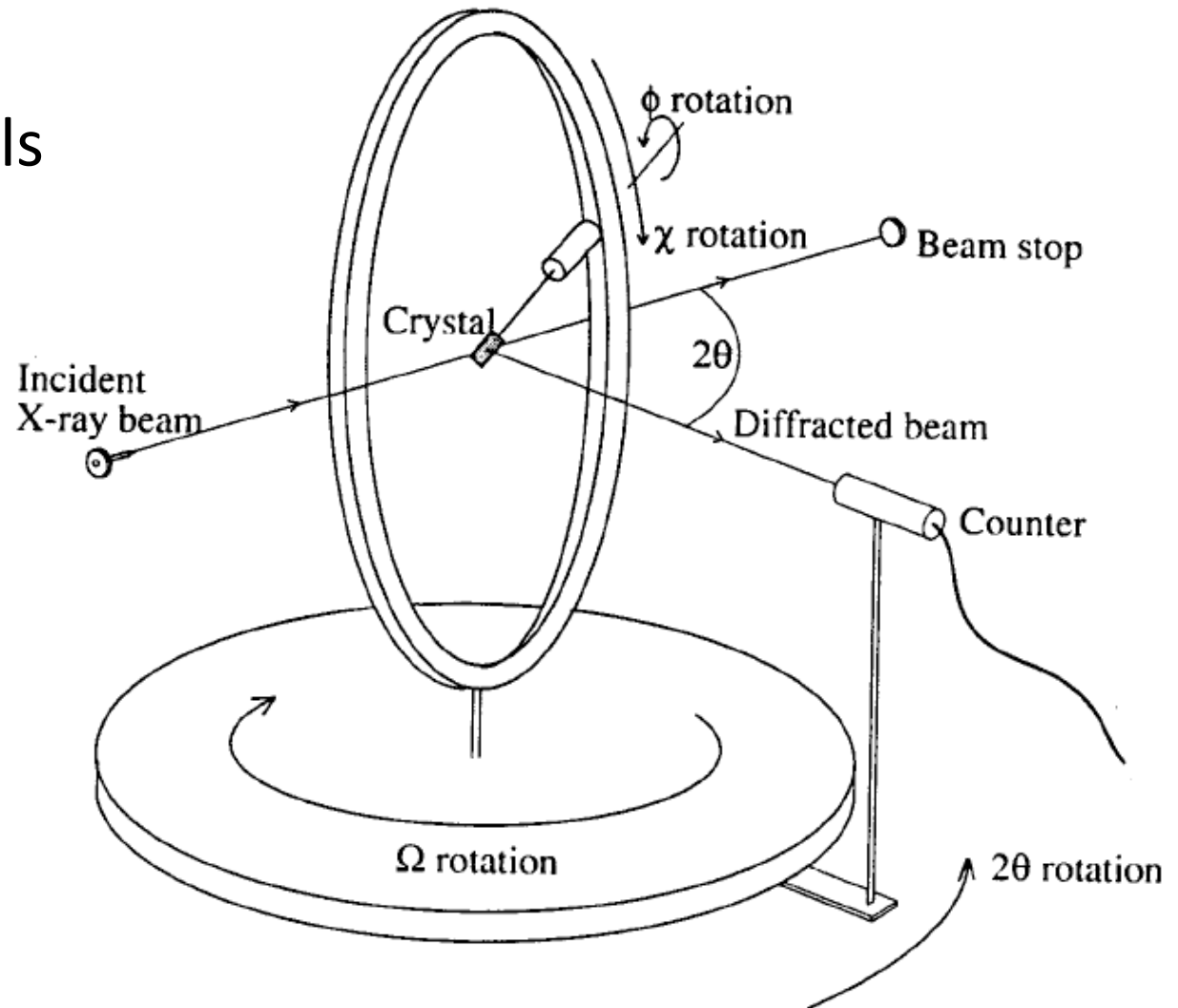
Two Circle Diffractometer

- For polycrystalline Materials



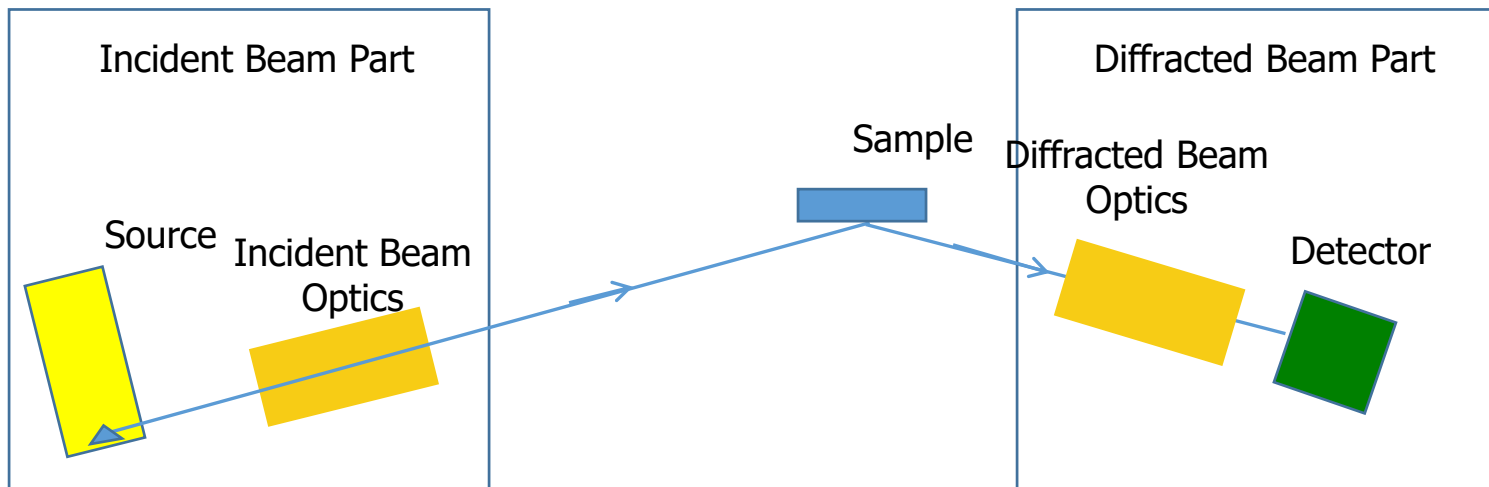
Four Circle Diffractometer

For single crystals

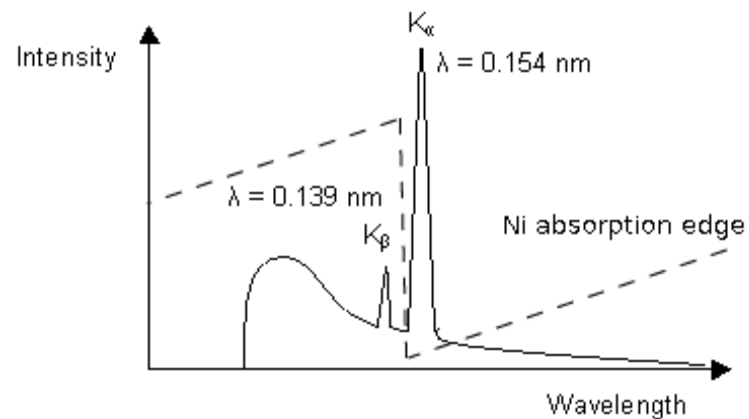


Geometry and Configuration

- **Theta-Theta** → Source and detector move θ , sample fixed
- **Theta-2Theta** → Sample moves θ and detector 2θ , source fixed



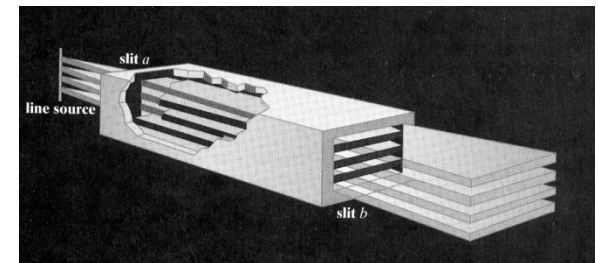
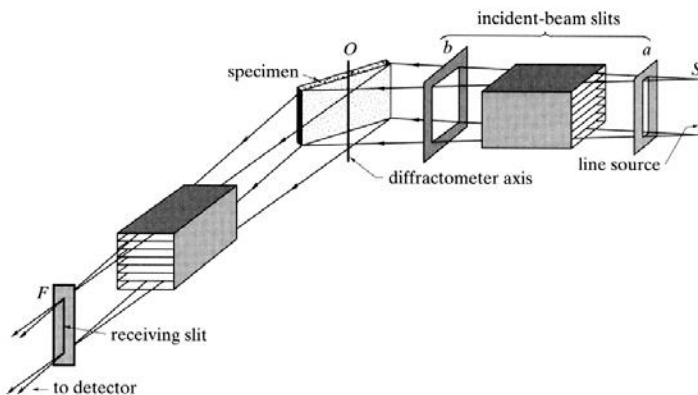
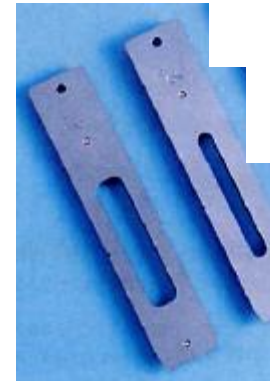
- Filter to remove K_β For eg. Ni filter for Cu K_β
- Reduction in intensity of K_α
- Choice of proper thickness

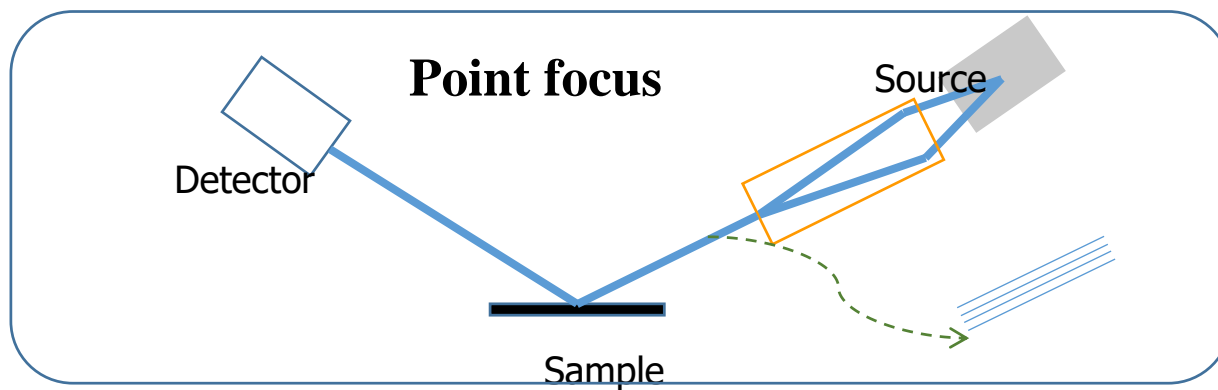
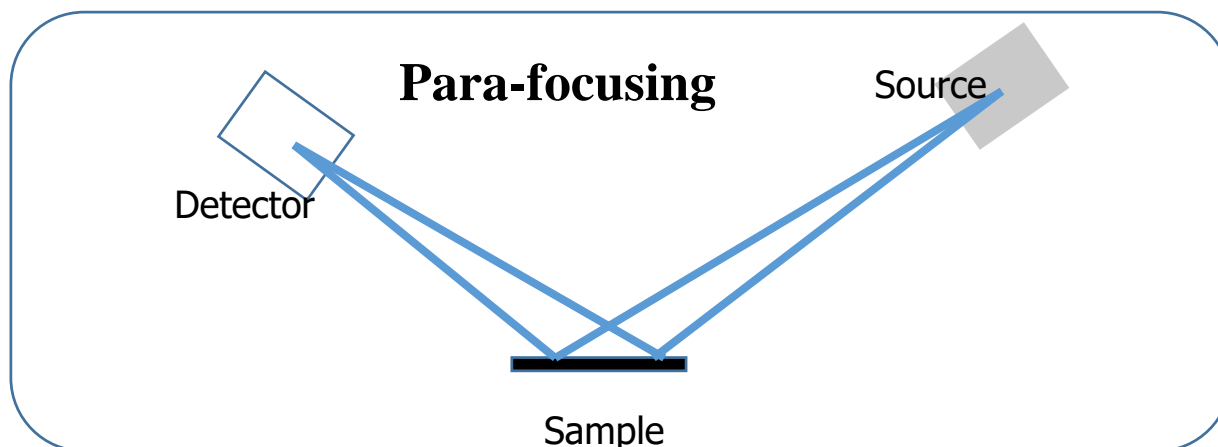
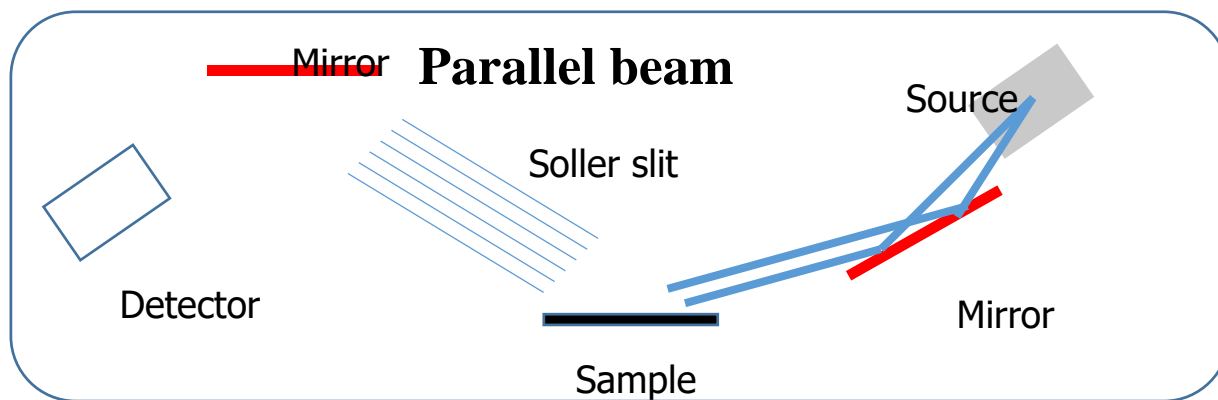


- Slits to limit the size of beam (Divergence slits)

To alter beam profile
(Soller slit angular divergence)

- Narrow slits = Lower intensity
+
Narrow peak

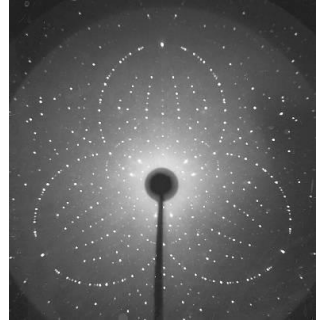




Comparison

Parallel beam	Para-focusing
X-rays are aligned	X-rays are diverging
Lower intensity for bulk samples	Higher intensity
Higher intensity for small samples	Lower intensity
Instrumental broadening independent of orientation of diffraction vector with specimen normal	Instrumental broadening dependent of orientation of diffraction vector with specimen normal
Suitable for GI-XRD	Suitable for Bragg-Brentano
Texture, stress	Powder diffraction

Diffraction Methods

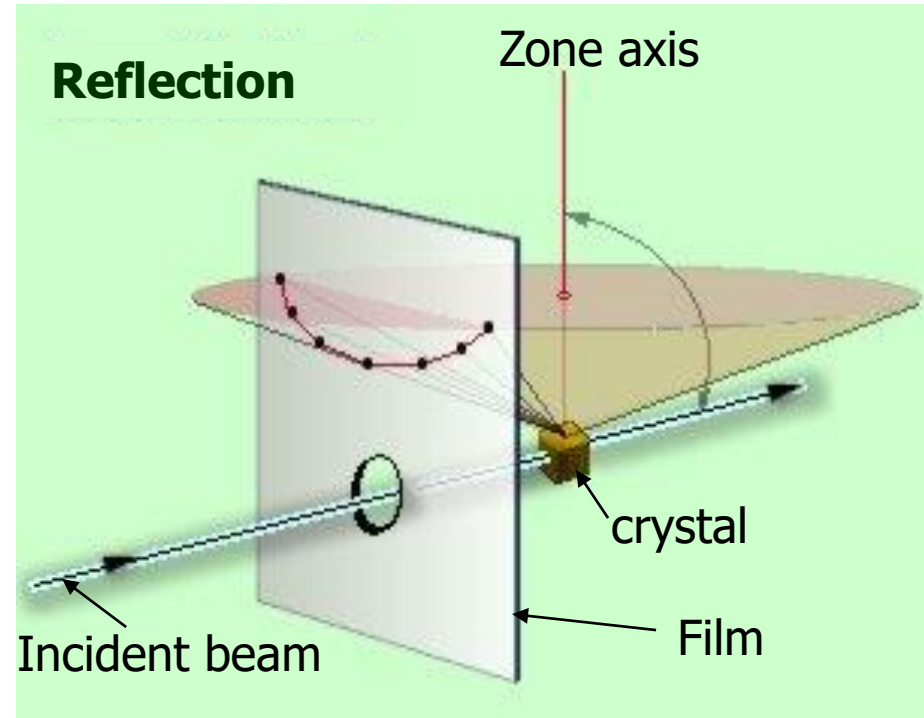
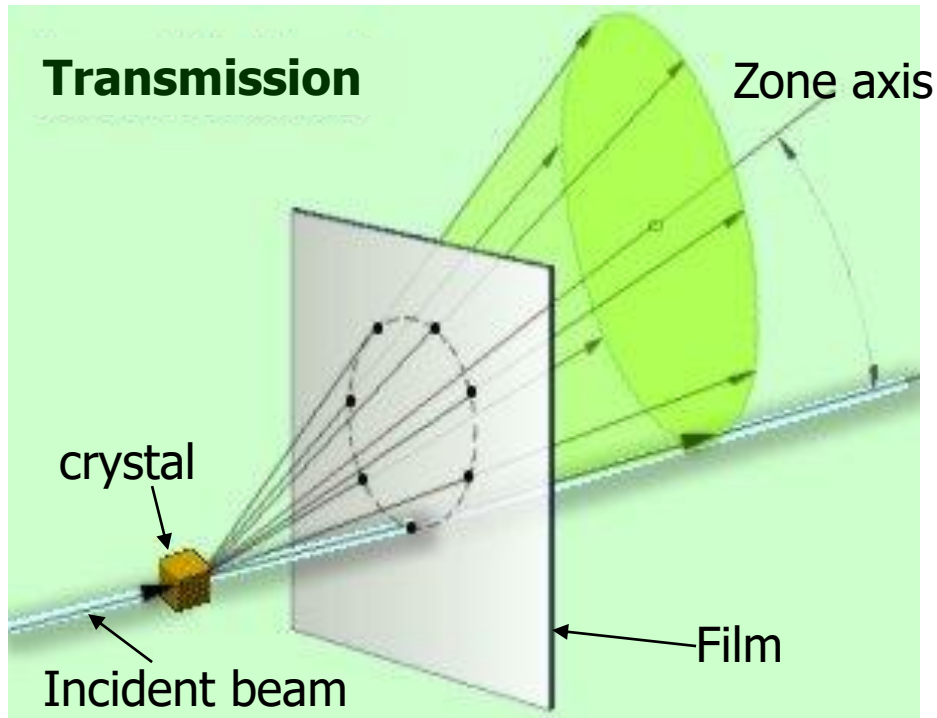


1. Laue method: a single crystal is held stationary in a beam of monochromatic x-ray radiation. The crystal diffracts the discrete values of l for which $\{hkl\}$ planes exist of spacing d_{hkl} and incidence angle θ . Used to determine symmetry of a crystal.
2. Rotating-crystal method: a single crystal is rotated about a fixed axis in a beam of monochromatic x-rays. The variation in θ brings different atomic planes into position for reflection.
3. Powder (Debye-Scherrer-Hull) method: a finely powdered sample is placed in a holder in a monochromatic x-ray beam, with the angle θ gradually changing due synchronous movement of holder and detector. Assuming random orientation of the tiny crystallites, there will be diffraction off of different $\{hkl\}$ planes at specific angles.

Diffraction Methods

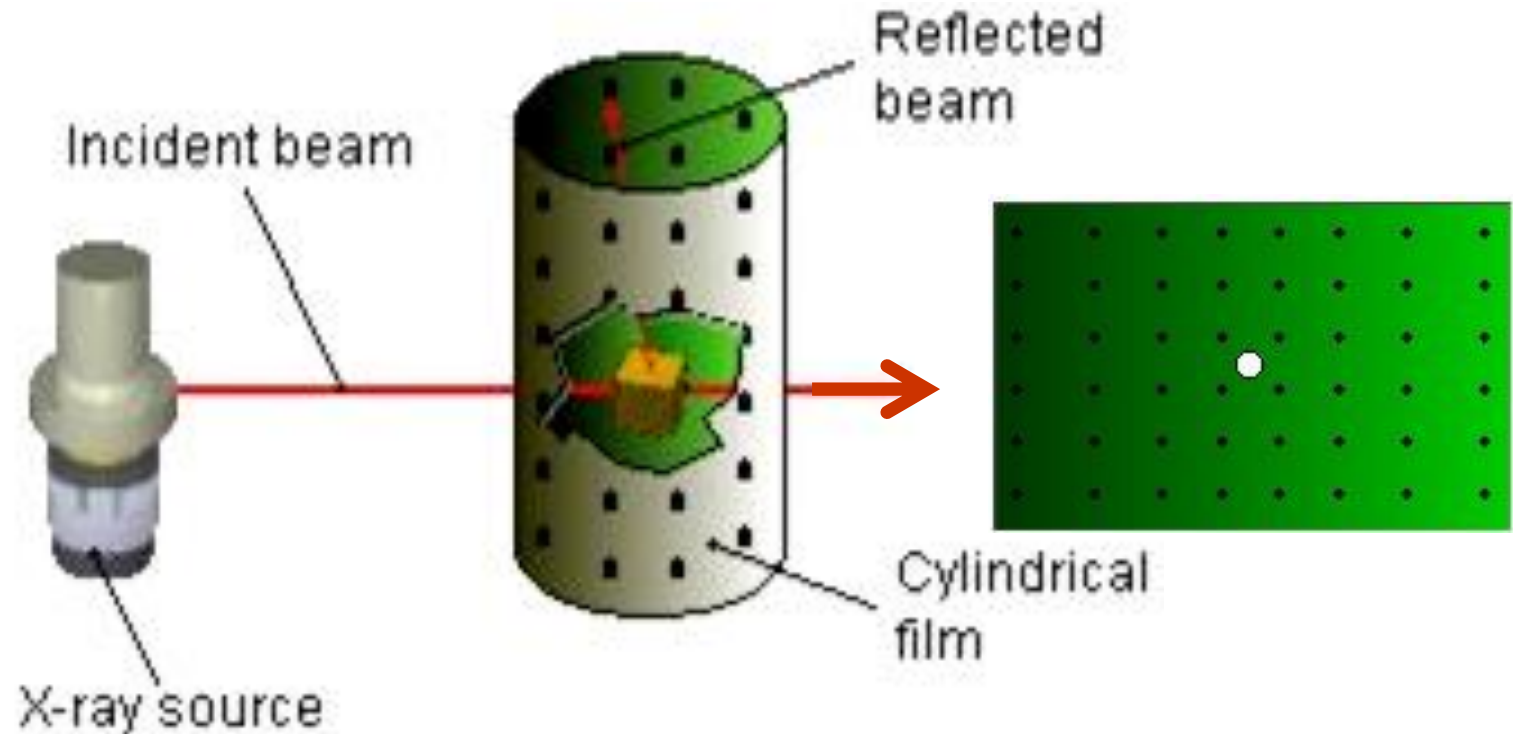
Method	Wavelength	Angle	Specimen
Laue	Variable	Fixed	Single Crystal
Rotating Crystal	Fixed	Variable (in part)	Single Crystal
Powder	Fixed	Variable	Powder

Laue Method



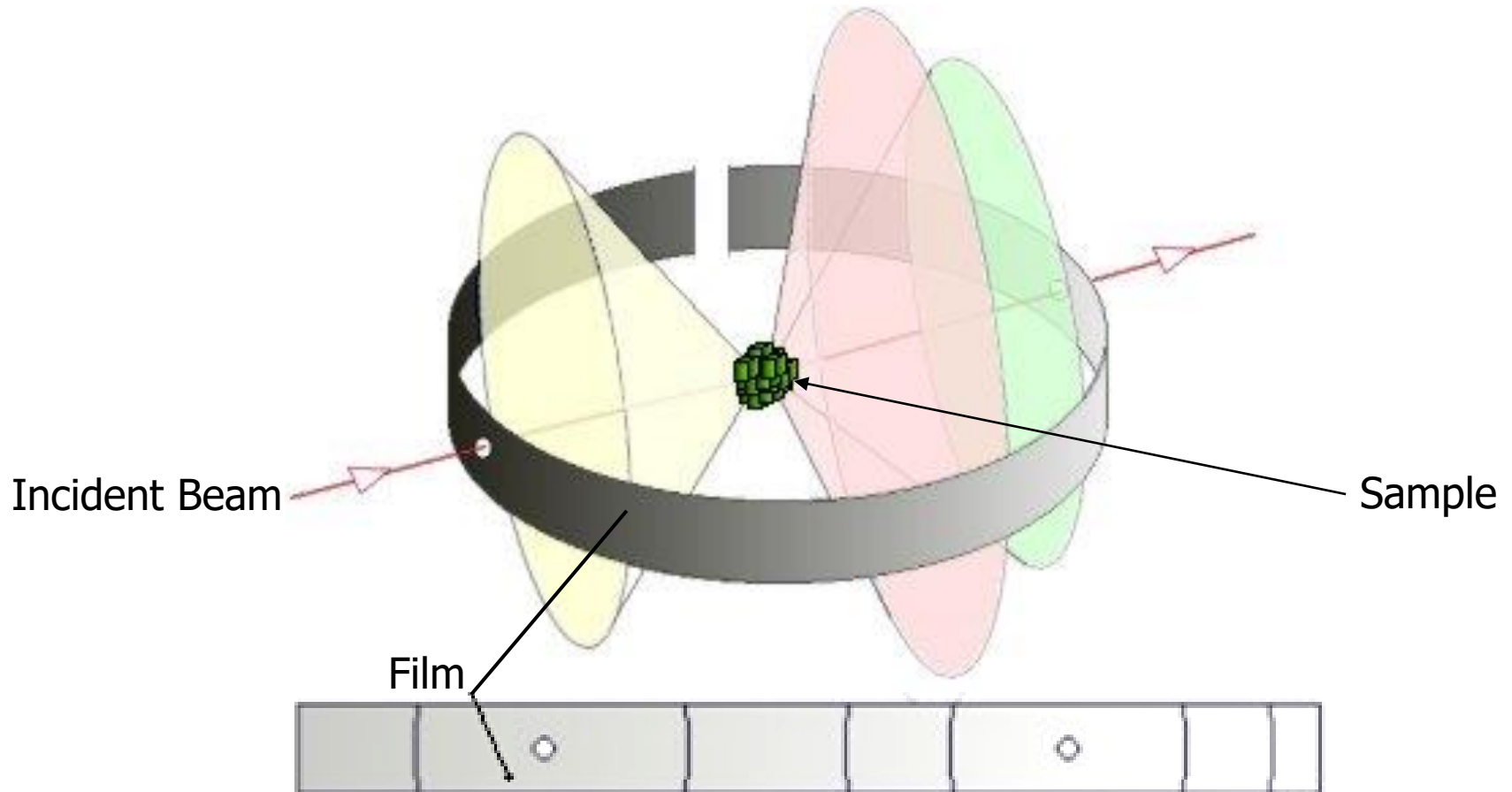
- Uses Single crystal
- Uses White Radiation
- Used for determining crystal orientation and quality

Rotating Crystal Method



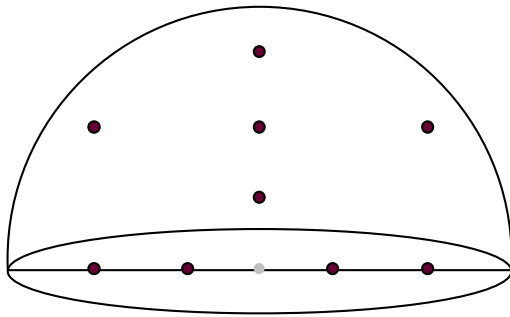
- Used for determination of unknown crystal structures

Powder Method

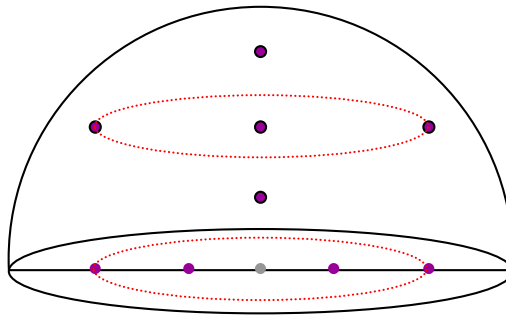
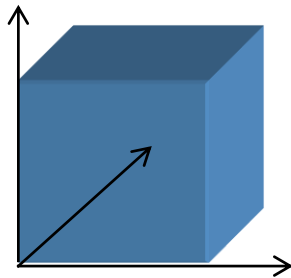


- Useful for determining lattice parameters with high precision and for identification of phases

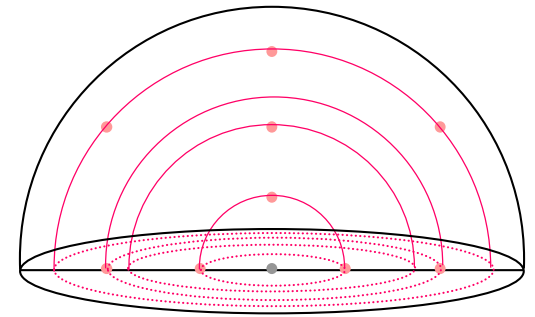
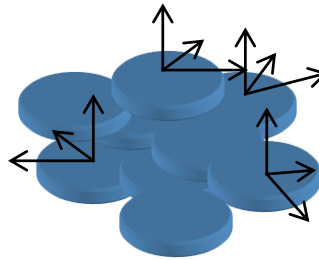
What does a powder really mean?



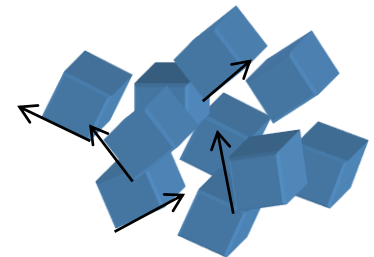
Single



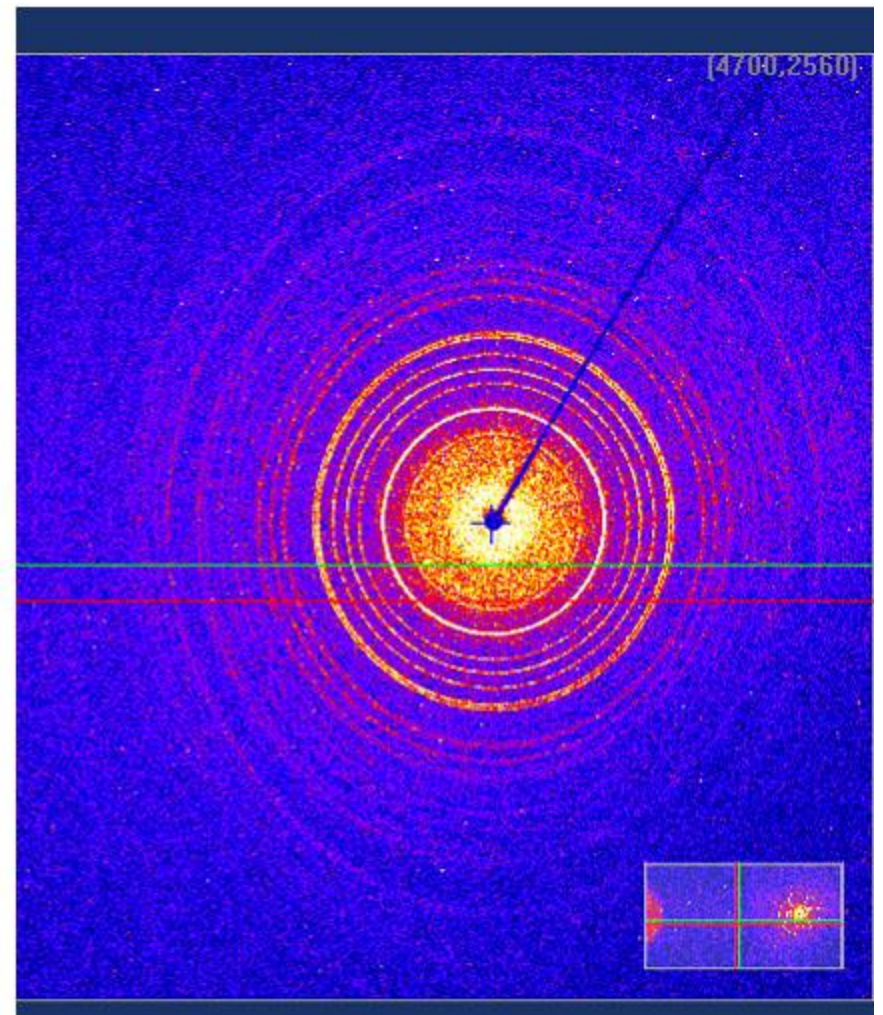
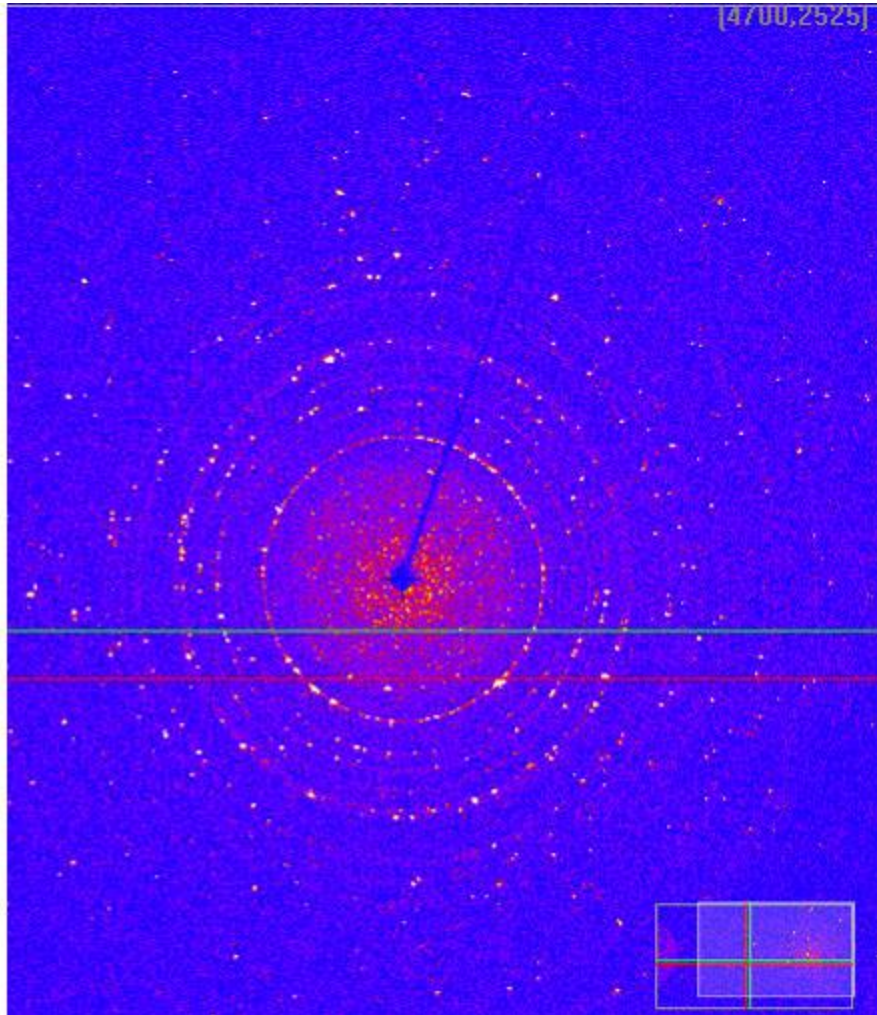
Oriented



Random

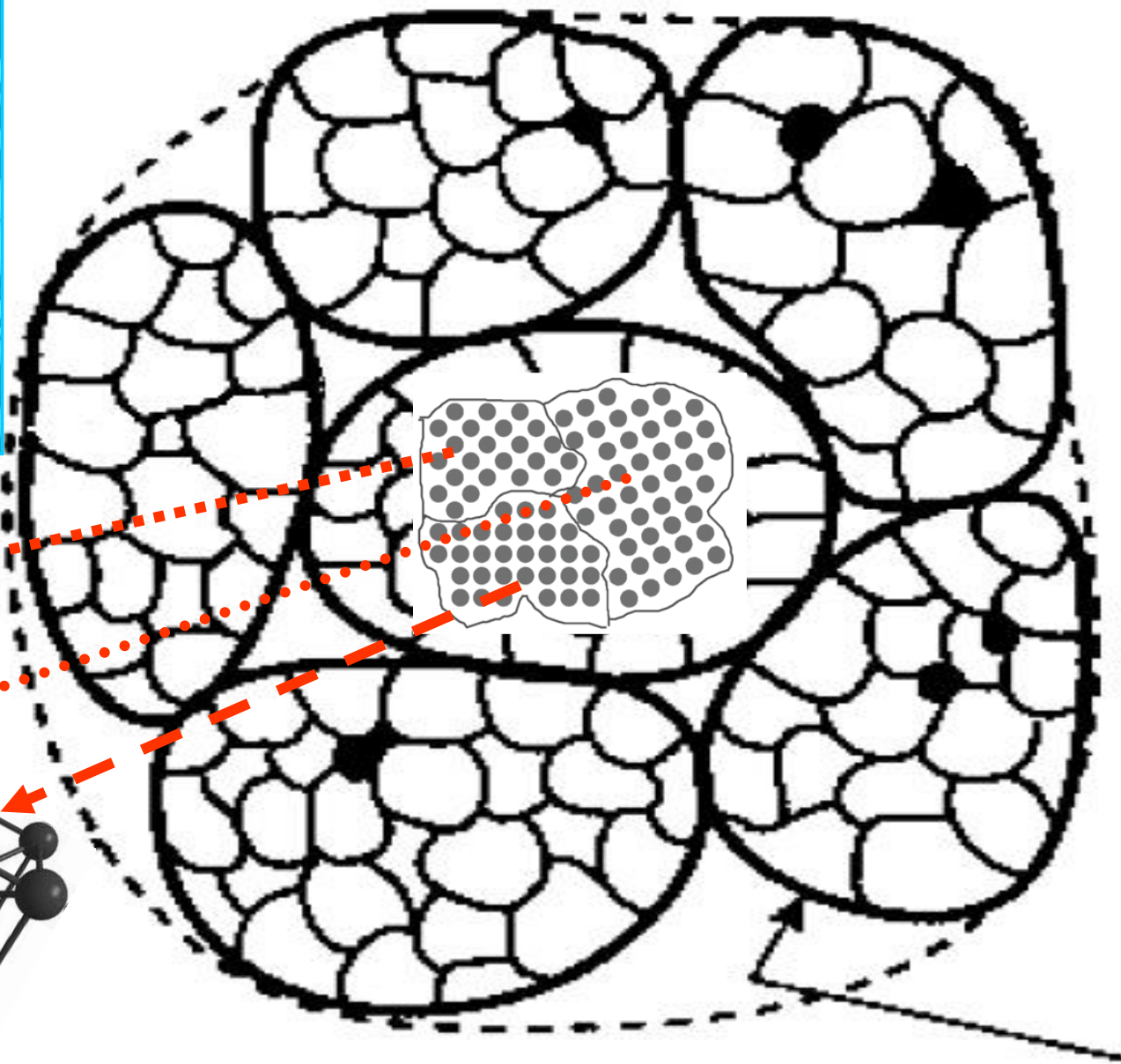
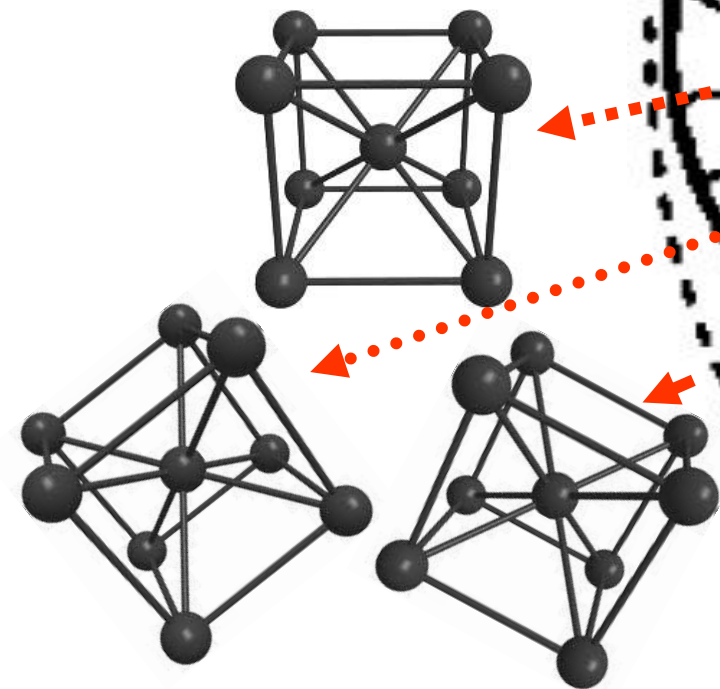
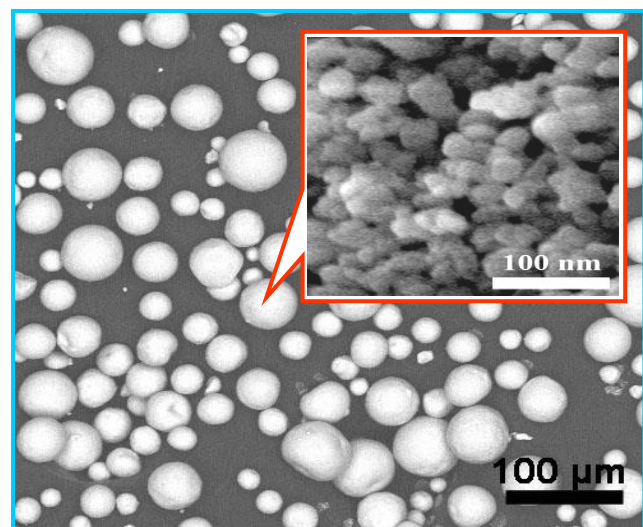


Limited crystals vs. many crystals

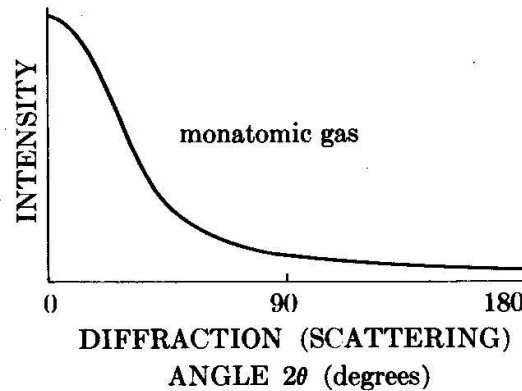
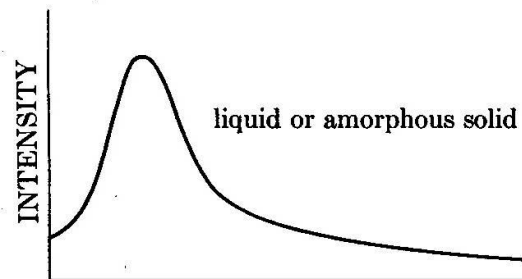
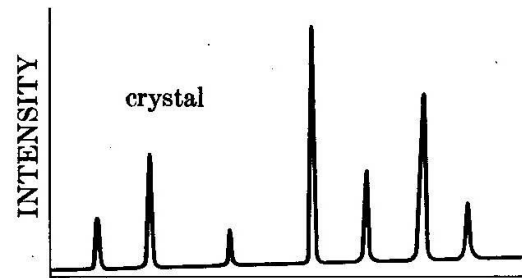


X-Ray Powder Diffraction

- Each crystallite in the powder is a small single crystal with a random orientation
- There is roughly equal statistically relevant number of crystallites for every set of randomly oriented planes that will diffract the incident beam
- Samples can be powder, sintered pellets, coatings on substrates, engine block
- The resultant XRD pattern is a unique fingerprint of the polycrystalline material which contains a lot of information
- The position, intensity, width, and shape of the observed diffraction peaks tells us about the crystal structure and, in some cases, microstructure of the sample



Diffraction from a variety of materials



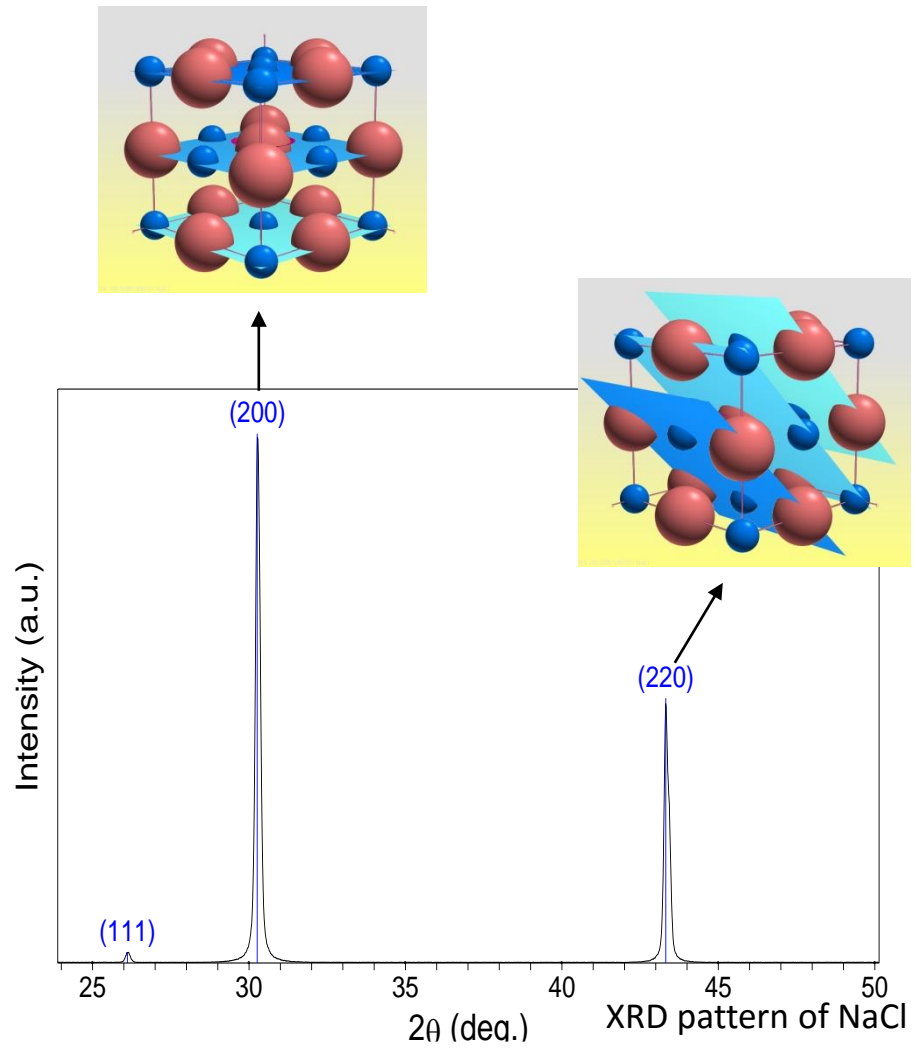
(From "Elements of X-ray Diffraction", B.D. Cullity, Addison Wesley)

What can be done with XRD?

- Identify phase composition
- Measure unit cell lattice parameters
- Estimate crystallite size, microstrain
- Measure residual stress
- Measure texture and/or epitaxy
- Evaluate thin film quality
- Measure multilayer thin film thickness, roughness, and density
- Determine orientation of single crystals
- Solve or refine crystal structures
- Analyze ordered meso- and nanostructures

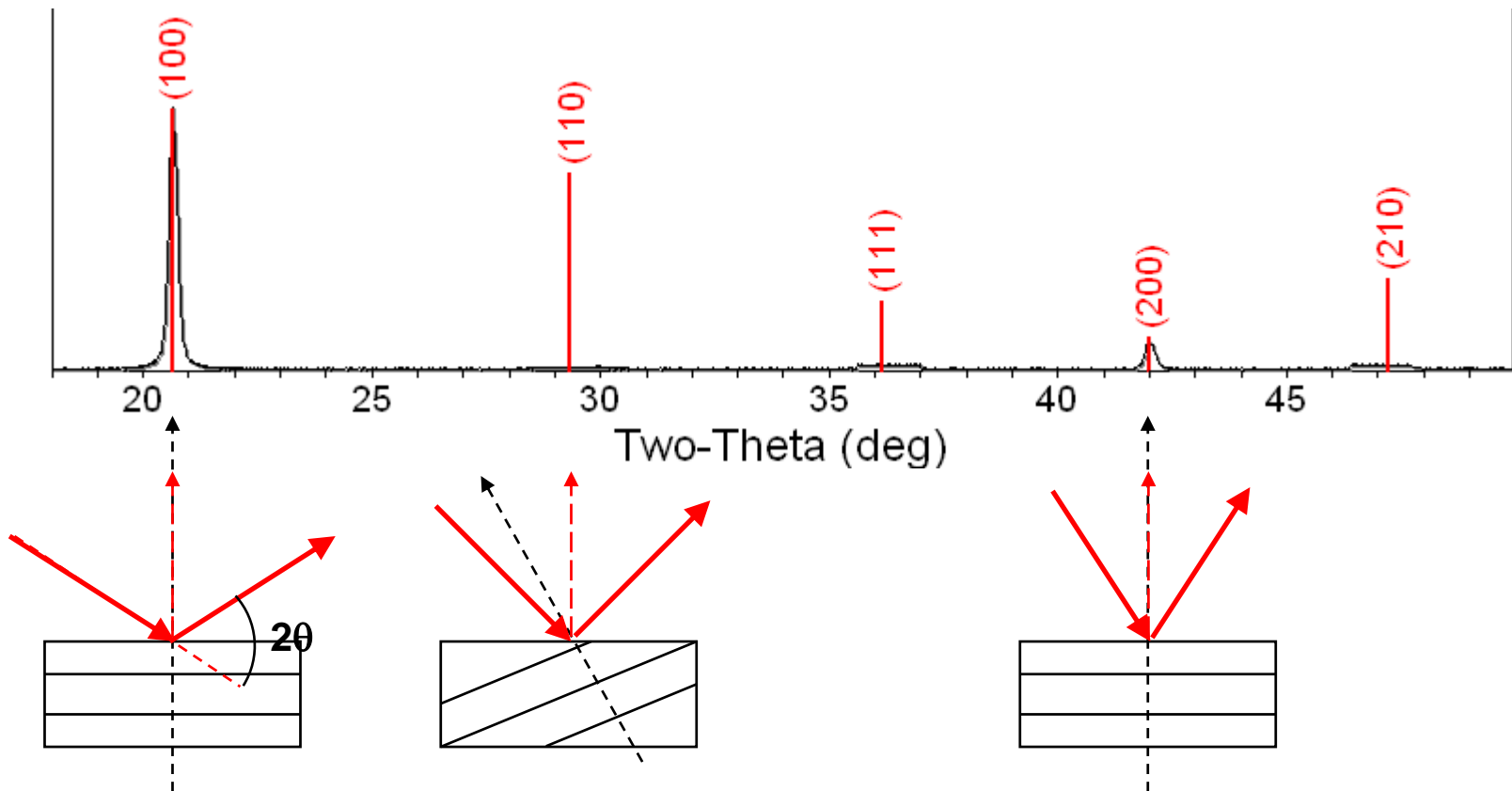
Diffraction Pattern

- Diffraction peak positions and intensity contain information about the crystal structure of a material
- Each diffraction peak is produced by a family of atomic planes.
- The position of the diffraction peak indicates the length d_{hkl} between those atomic planes
 - this ultimately correlates to the bond distances between atoms
- The intensity of the diffraction peak is determined by what atoms are present on the diffracting plane, their scattering factors, and the structure factor



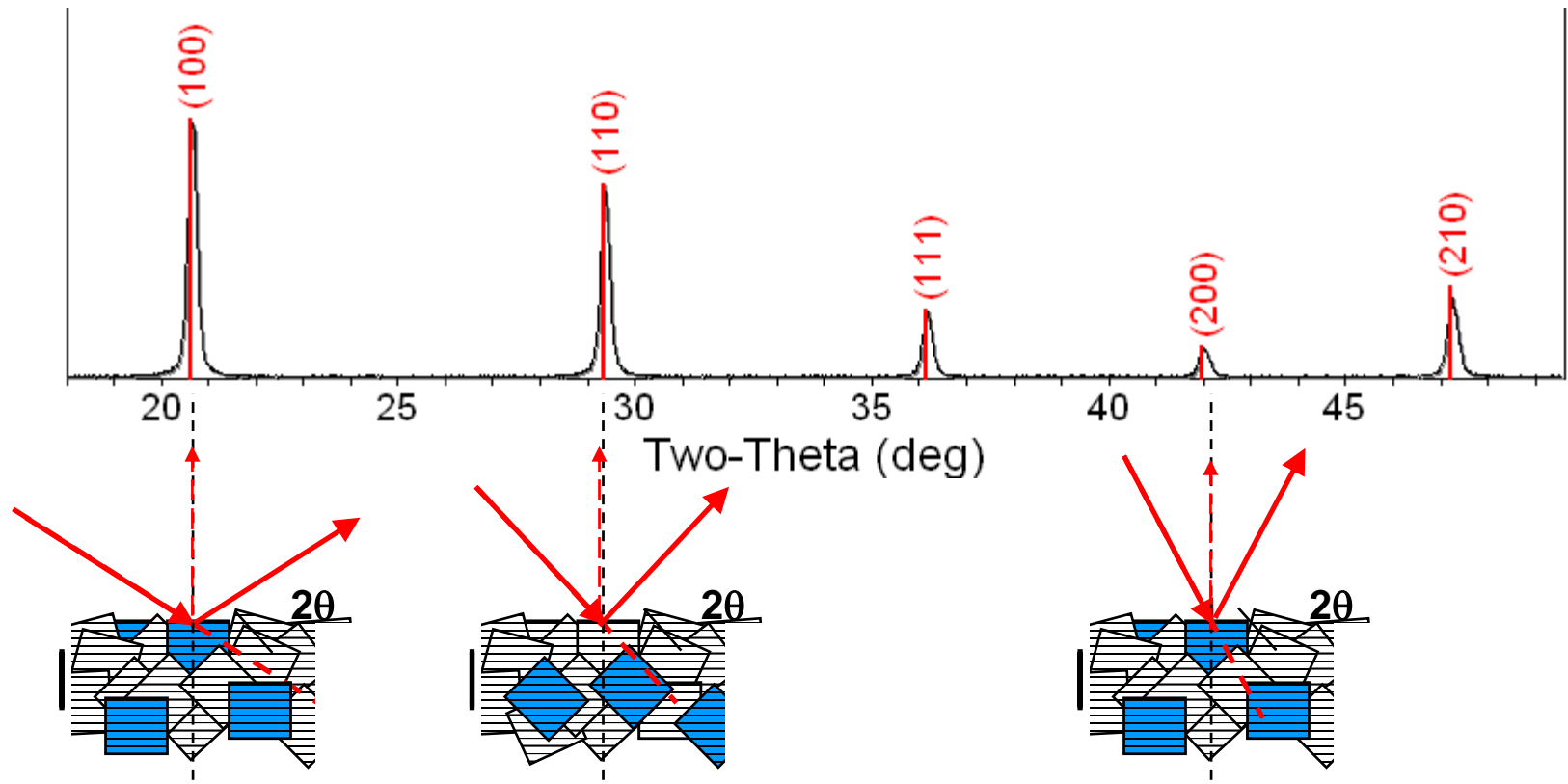
Diffraction of a Single Crystal

- A single crystal in a Bragg-Brentano diffractometer produces one family of peaks in the diffraction pattern



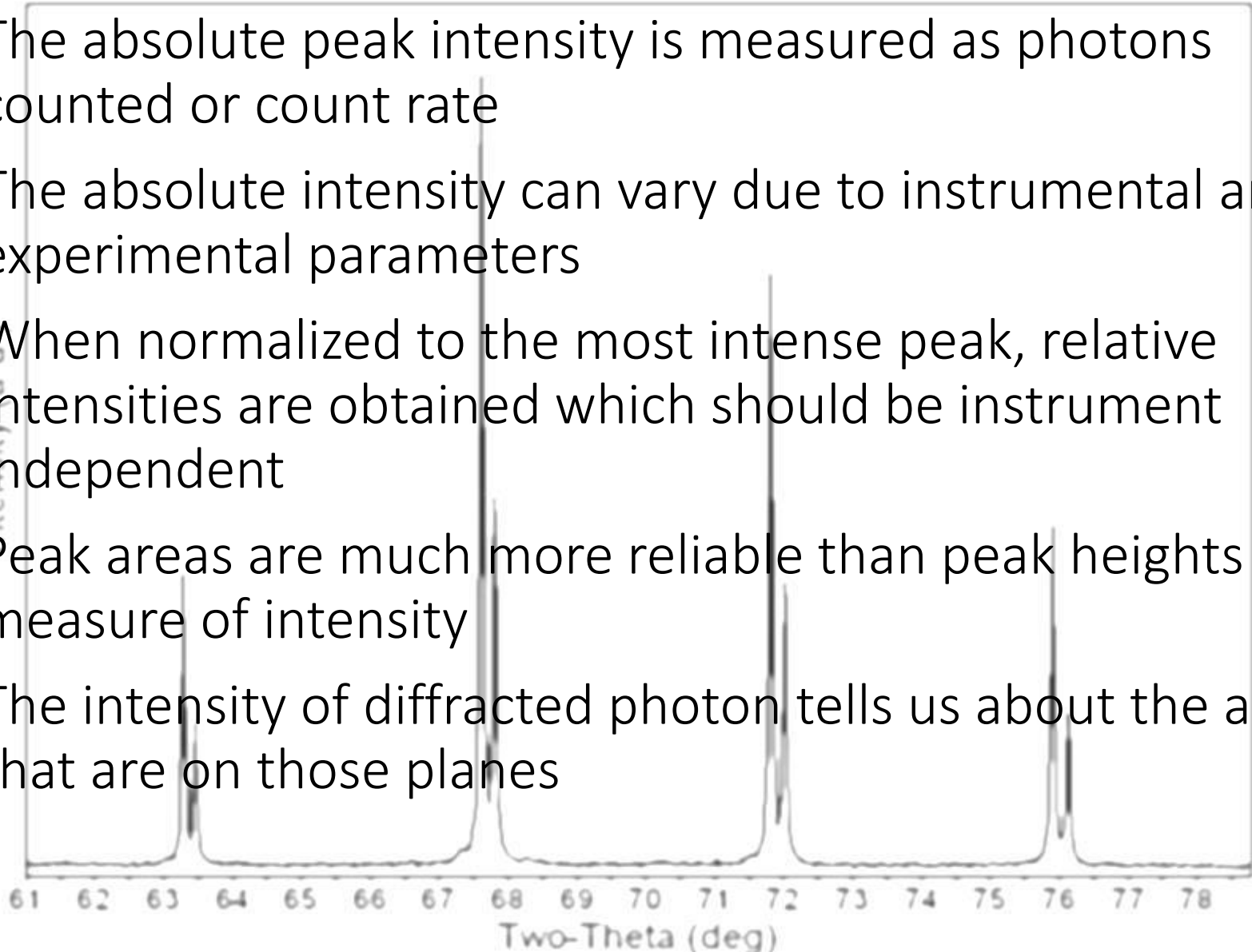
Diffraction of a Polycrystalline Sample

- A polycrystalline sample contains thousands of crystallites
- All diffraction peaks should be observed
- There will be a small percentage of crystallites that are properly oriented to diffract for every set of planes

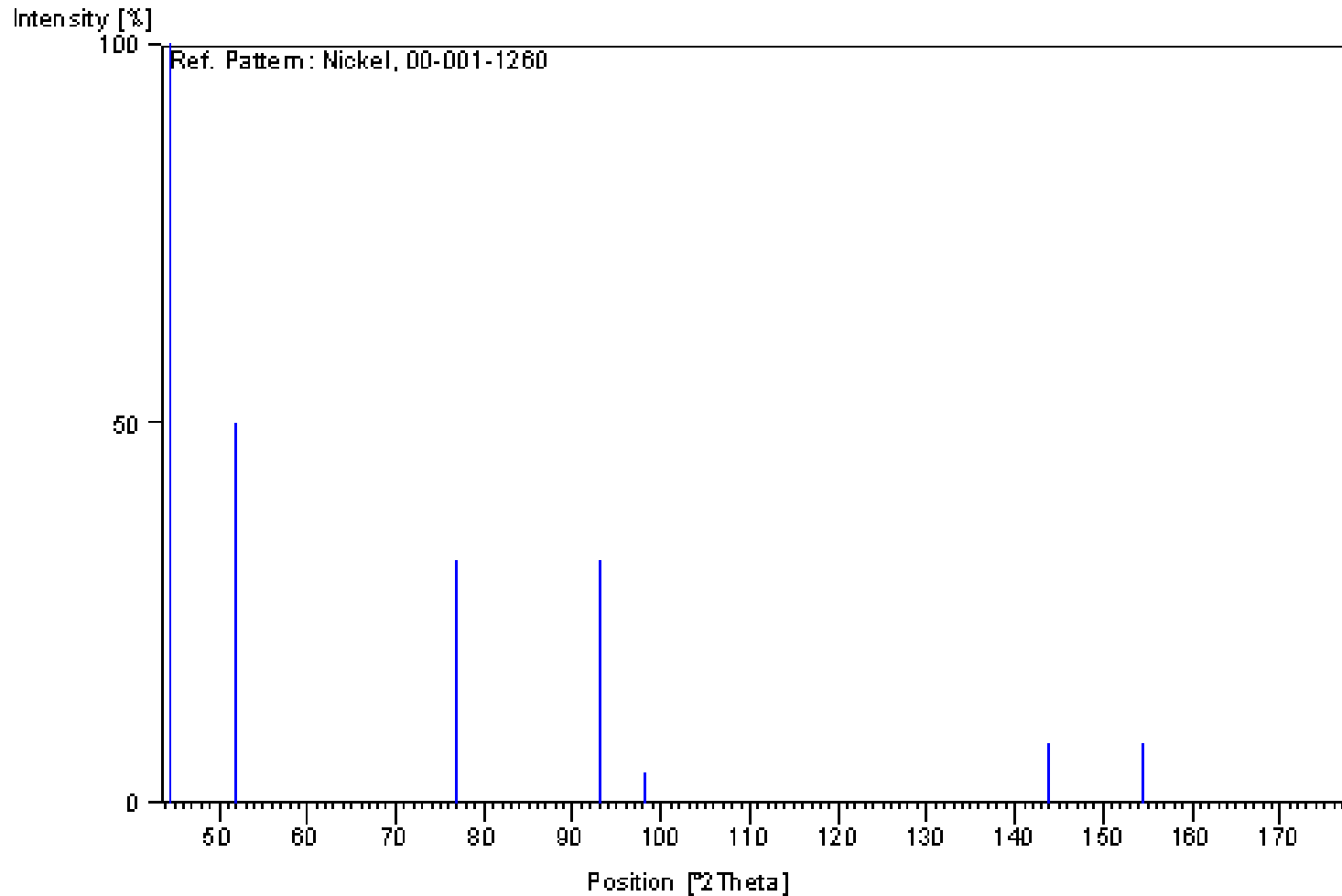


Diffraction Peaks

- The absolute peak intensity is measured as photons counted or count rate
- The absolute intensity can vary due to instrumental and experimental parameters
- When normalized to the most intense peak, relative intensities are obtained which should be instrument independent
- Peak areas are much more reliable than peak heights as a measure of intensity
- The intensity of diffracted photon tells us about the atoms that are on those planes

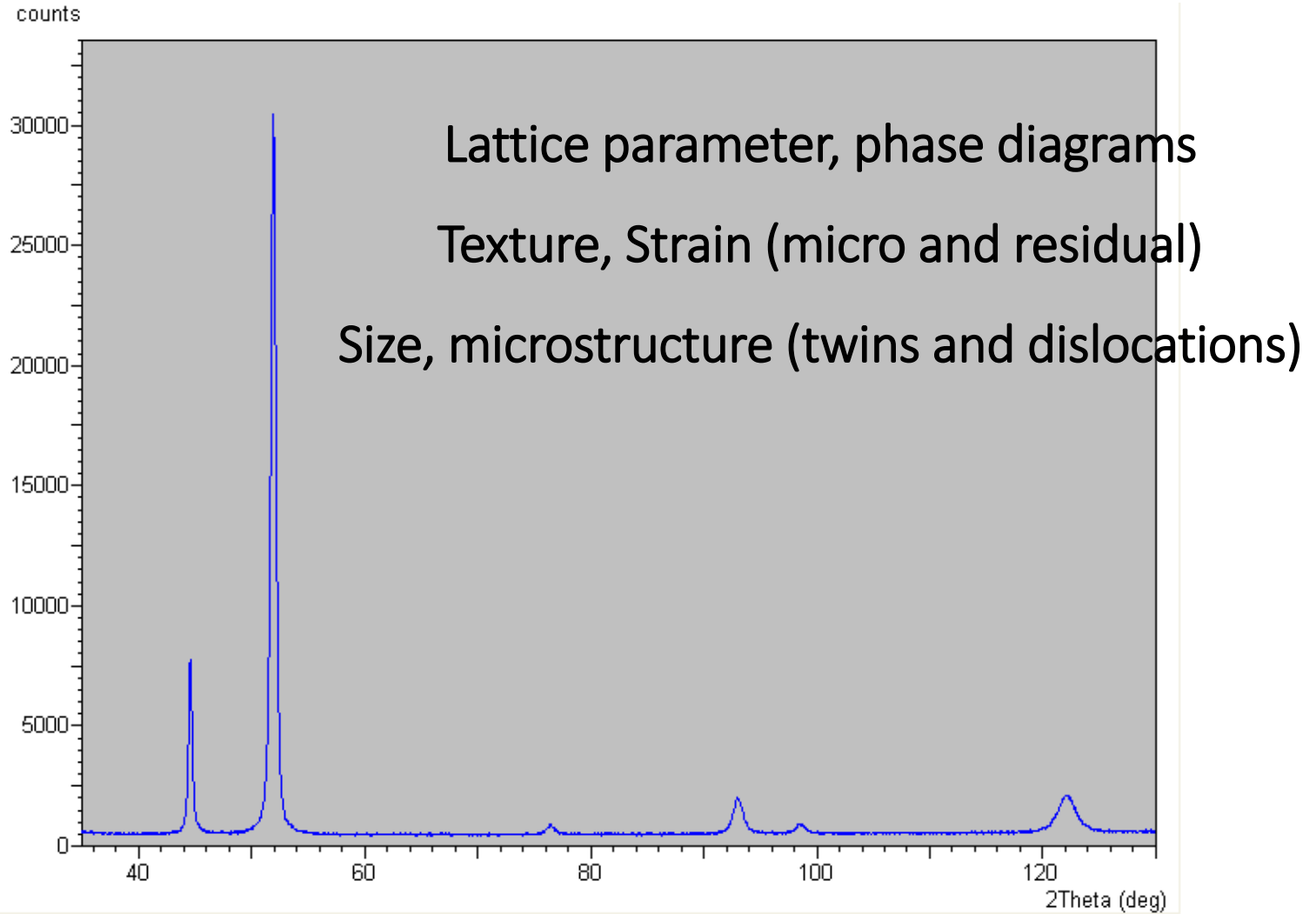


Stick pattern from JCPDS



<http://ww1.iucr.org/cww-top/crystal.index.html>

Actual Pattern



Bulk electrodeposited nanocrystalline nickel

Qualitative Phase Identification

- Mostly used for analysis of processes and quality control

When

You found something in the laboratory and want to know what it is

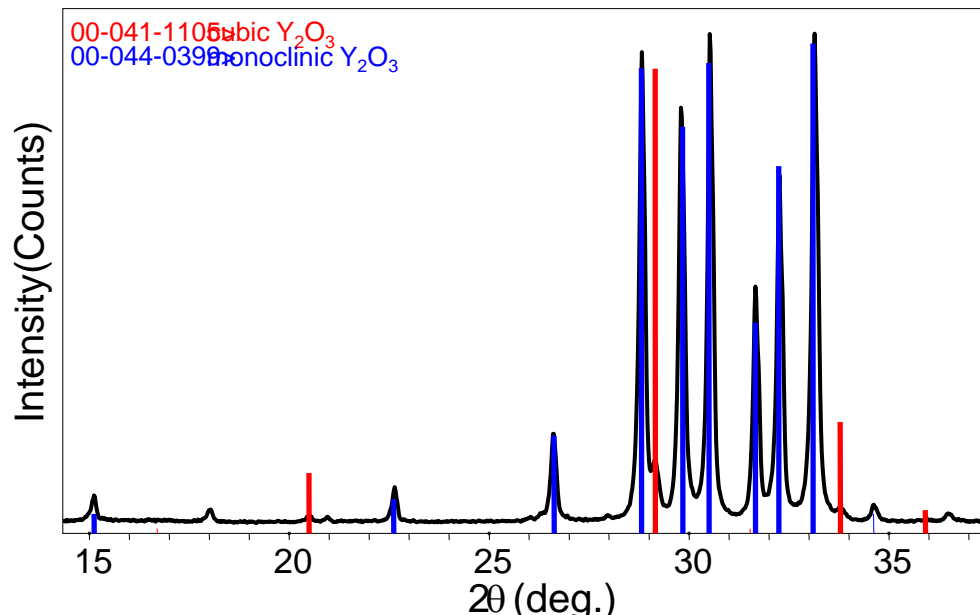
or

You tried to synthesize a material and want to know if you really did

or

You want to check if your raw materials are pure

Example: You have some monoclinic Y_2O_3 in the lab for a long time and want to see if it degraded in time

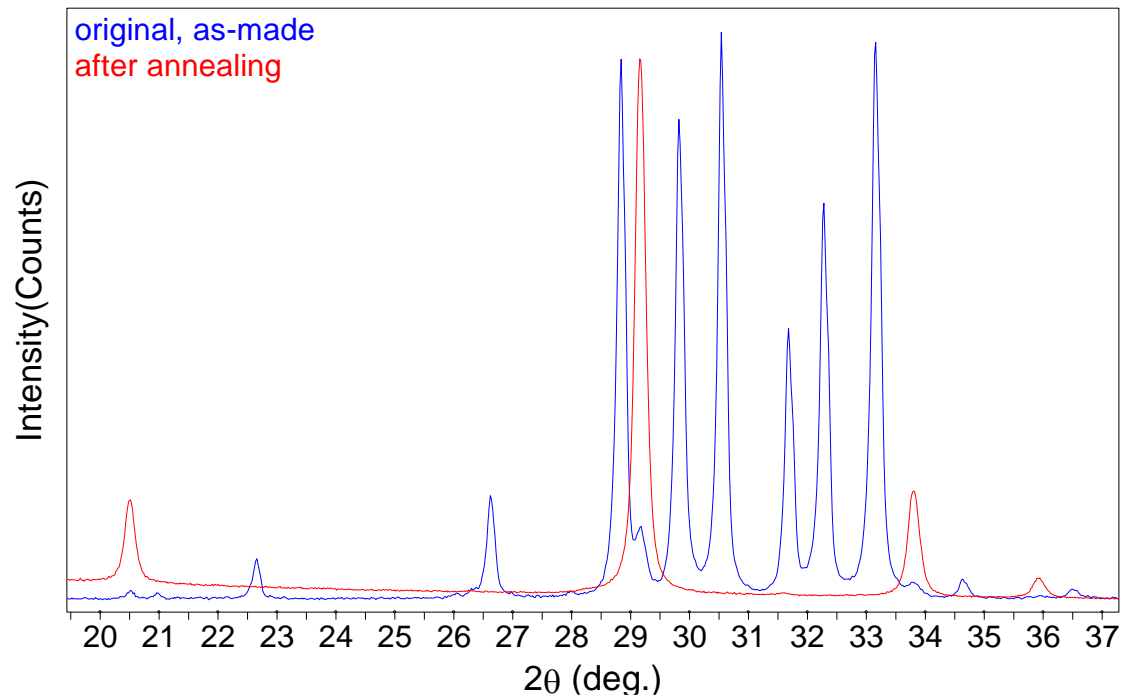


Qualitative Phase Identification can be used to study reactions *ex situ*

- Comparing and identifying the XRD patterns of a material before and after a phase changing reaction

Example: You heated the monoclinic Y_2O_3 to 1000 °C and want to see the level of degradation

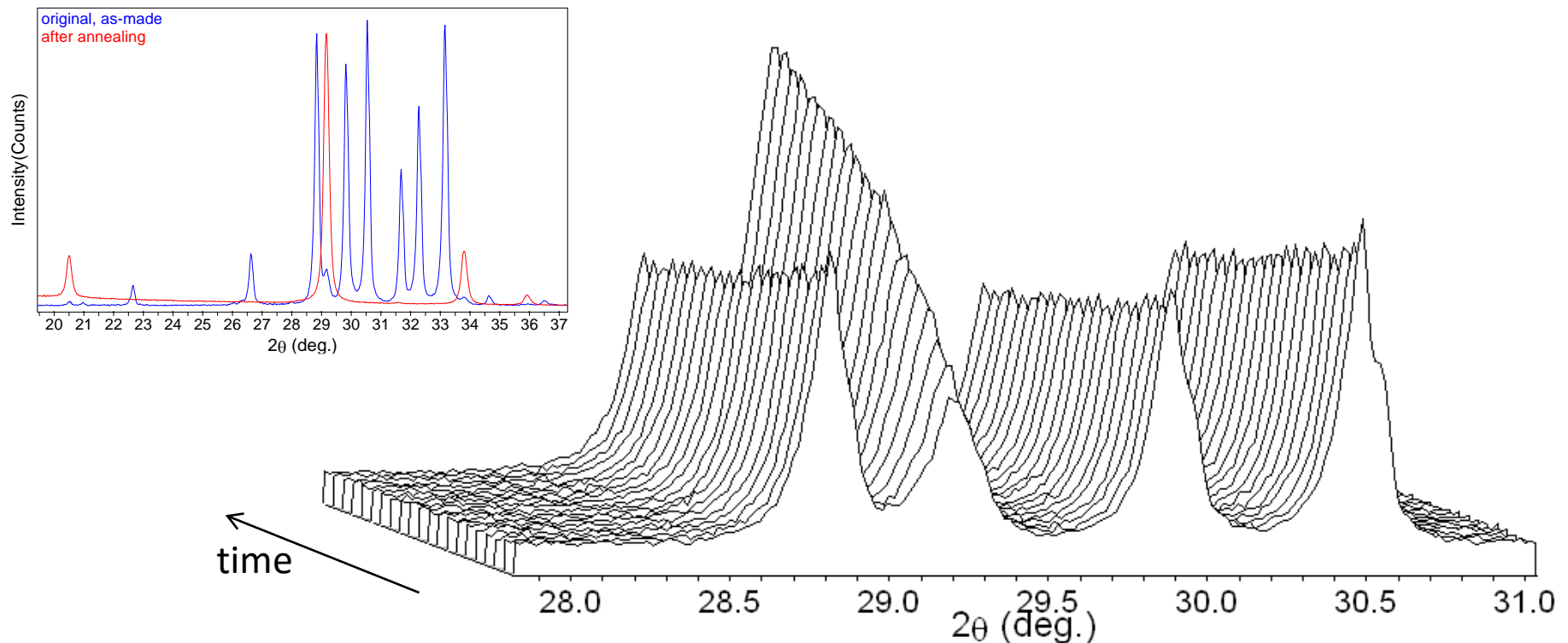
Heat treatment converted all of the monoclinic Y_2O_3 into cubic Y_2O_3



Qualitative Phase Identification can also be used to study reactions *in situ*

- A phase changing reaction such as heat treatment is monitored directly by the attached X-Ray diffractometer

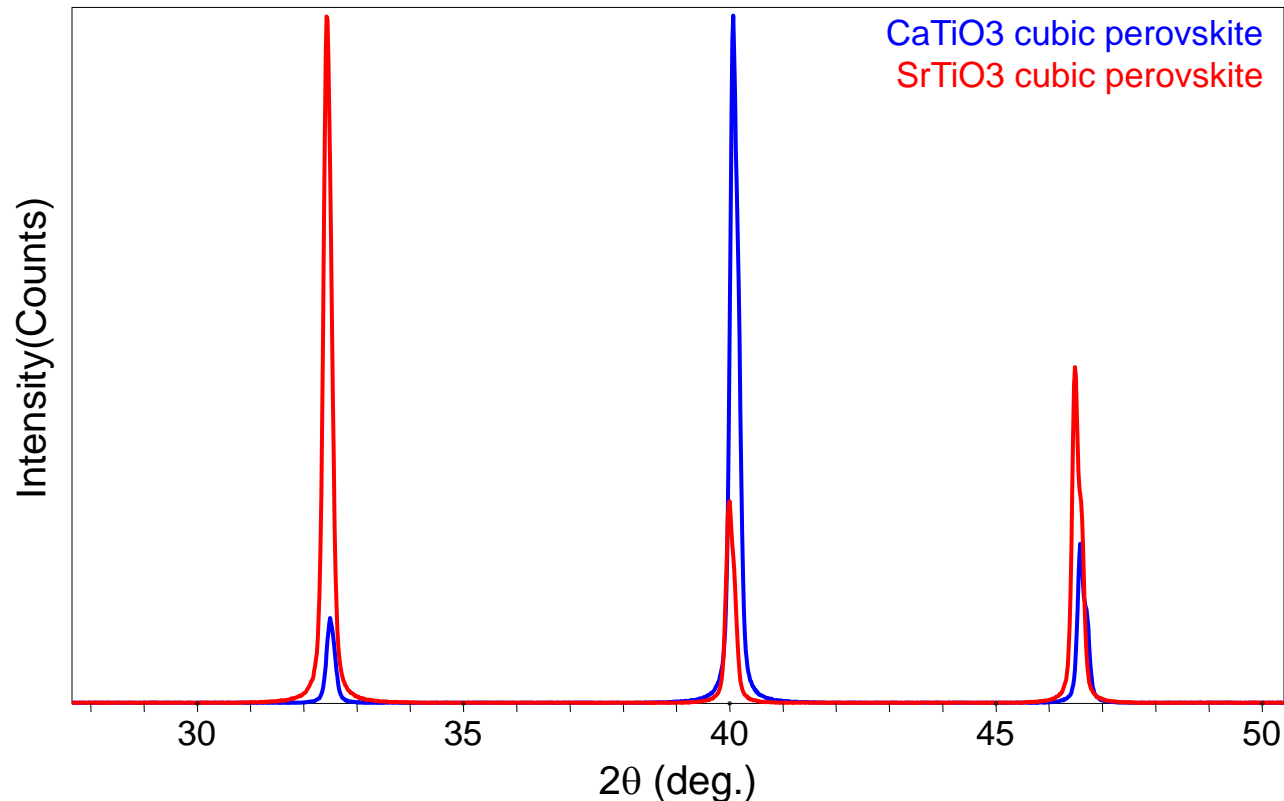
Example: You heat the monoclinic Y_2O_3 to 1000 °C and want to see the rate of degradation



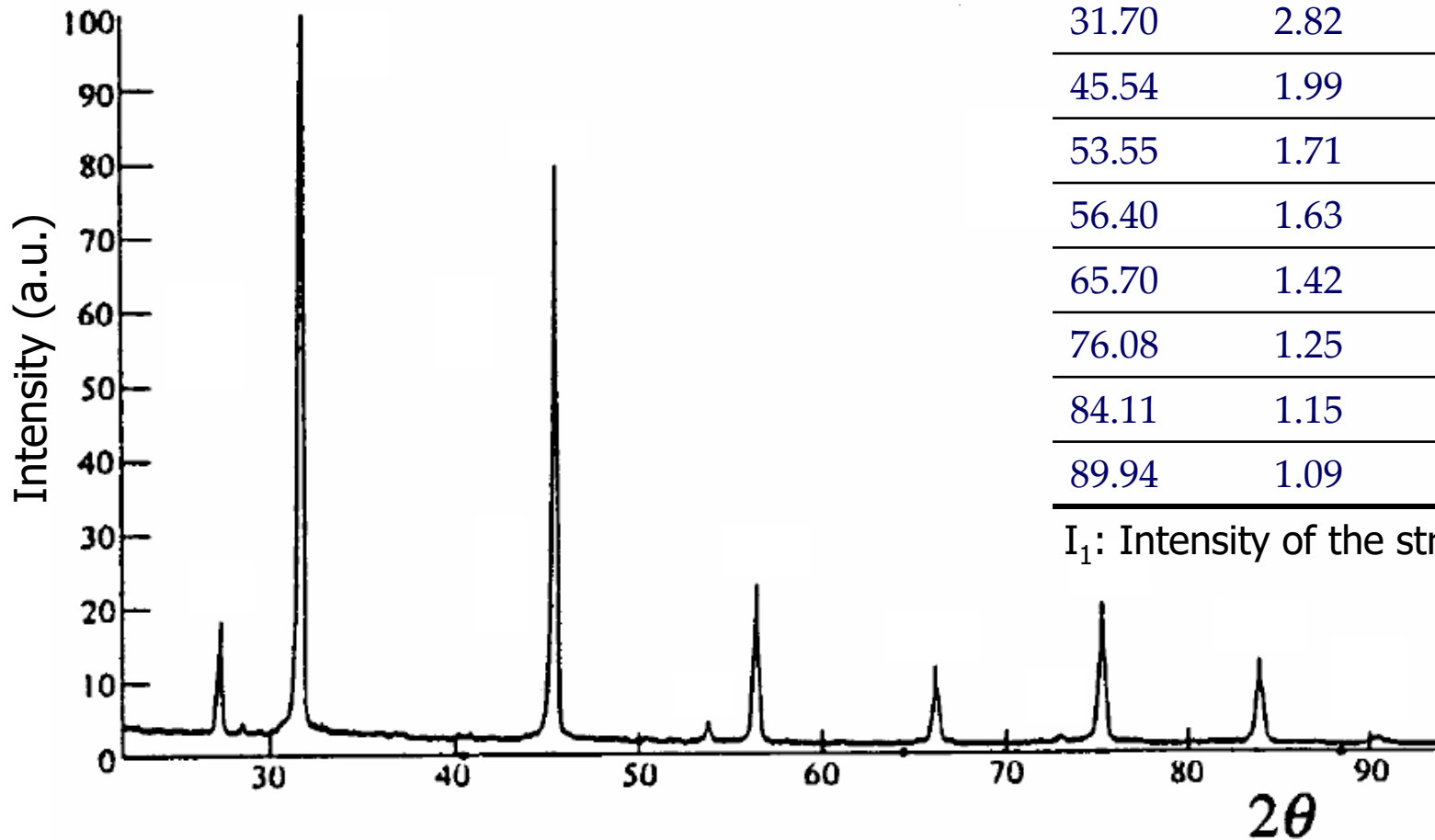
Qualitative Phase Identification can also be used to discern between isostructural compounds and polymorphs

- Differences in electron density can be mapped because X-Rays scatter proportionally to Z^2

Example: the cubic phases of CaTiO_3 and SrTiO_3 have identical crystal structures, with the cation replaced by Ca or Sr respectively



Analysis of Single Phase



$2\theta(^{\circ})$	$d\text{ (\AA)}$	$(I/I_1)*100$
27.42	3.25	10
31.70	2.82	100
45.54	1.99	60
53.55	1.71	5
56.40	1.63	30
65.70	1.42	20
76.08	1.25	30
84.11	1.15	30
89.94	1.09	5

I_1 : Intensity of the strongest peak

Procedure

- Note first three strongest peaks at d_1 , d_2 , and d_3
- In the present case: d_1 : 2.82; d_2 : 1.99 and d_3 : 1.63 Å
- Search JCPDS manual to find the d group belonging to the strongest line: between 2.84-2.80 Å
- There are 17 substances with approximately similar d_2 but only 4 have d_1 : 2.82 Å
- Out of these, only NaCl has d_3 : 1.63 Å
- **It is NaCl**

Specimen and Intensities	Substance	File Number
2.82 ₉ 1.99 ₉ 2.26 _x 1.61 ₉ 1.51 ₉ 1.49 ₉ 3.57 ₈ 2.66 ₈	(ErSe) ₂ Q	19-443
2.82 _x 1.99 ₆ 1.63 ₂ 3.26 ₁ 1.26 ₁ 1.15 ₁ 1.41 ₁ 0.89 ₁	NaCl	5-628
2.82 ₄ 1.99 ₄ 1.54 _x 1.20 ₄ 1.19 ₄ 2.44 ₃ 5.62 ₂ 4.89 ₂	(NH ₄) ₂ WO ₂ Cl ₄	22-65
2.82 _x 1.99 ₈ 1.26 ₃ 1.63 ₂ 1.15 ₂ 0.94 ₁ 0.89 ₁ 1.41 ₁	(BePd)2C	18-225

Caution: It could be much more tricky if the sample is oriented or textured or your goniometer is not calibrated

Presence of Multiple phases

- More Complex
- Several permutations combinations possible
- e.g. d_1 ; d_2 ; and d_3 , the first three strongest lines show several alternatives
- Then take any of the two lines together and match
- It turns out that 1st and 3rd strongest lies belong to Cu and then all other peaks for Cu can be separated out

Pattern for Cu	
d (Å)	I/I ₁
2.088	100
1.808	46
1.278	20
1.09	17
1.0436	5
0.9038	3
0.8293	9
0.8083	8

d (Å)	I/I ₁
3.01	5
2.47	72
2.13	28
2.09 *	100
1.80 *	52
1.50	20
1.29	9
1.28 *	18
1.22	4
1.08 *	20
1.04 *	3
0.98	5
0.91	4
0.83 *	8
0.81 *	10

Presence of Multiple phases

- Now separate the remaining lines and normalize the intensities
- Look for first three lines and it turns out that the phase is Cu_2O
- If more phases, harder to solve

Pattern of Cu_2O	
d (Å)	I/I ₁
3.020	9
2.465	100
2.135	37
1.743	1
1.510	27
1.287	17
1.233	4
1.0674	2
0.9795	4

Remaining Lines		
d (Å)	I/I ₁	
	Observed	Normalized
3.01	5	7
2.47	72	100
2.13	28	39
1.50	20	28
1.29	9	13
1.22	4	6
0.98	5	7

Diffraction Data is Essential

PDF # 860740, Wavelength = 1.54060 (Å)

86-0740 Quality: C

CAS Number:

Molecular Weight: 502.32

Volume[CD]: 521.26

Dx: 3.200 Dm:

Sys: Hexagonal

Lattice: Primitive

S.G.: P6₃/m (176)

Cell Parameters:

a 9.352 b c 6.882

α β γ

I/I_{cor}: 1.04

Rad: CuKα1

Lambda: 1.54060

Filter:

d-sp: calculated

ICSD #: 081442

Mineral Name:

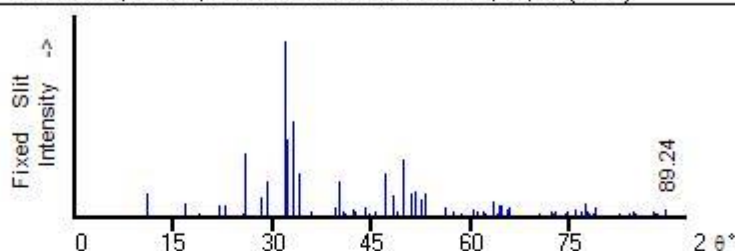
Hydroxylapatite

Ca₅(P₃O₄)₃(OH)

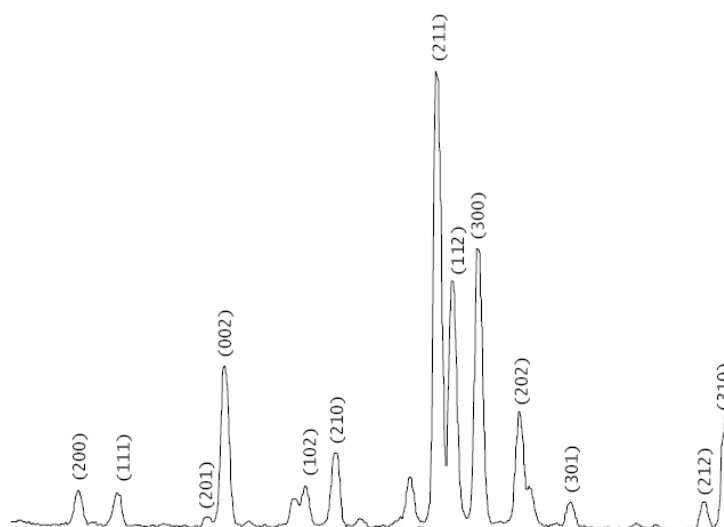
Calcium Phosphate Hydroxide

Ref: Calculated from ICSD using POWD-12++, (1997)

Ref: Tomita, K et al., J. Mineral. Petrol. Econ. Geol., 91, 11 (1996)



2θ	Int-f	h	k	l	2θ	Int-f	h	k	l	2θ	Int-f	h	k	l
10.915	125	1	0	0	45.402	32	2	0	3	63.950	17	5	1	0
16.892	69	1	0	1	46.947	248	2	2	2	64.134	64	3	0	4
18.964	22	1	1	0	48.350	116	1	3	2	64.485	58	3	2	3
21.931	66	2	0	0	48.985	33	2	3	0	65.537	44	5	1	1
22.976	64	1	1	1	49.620	323	2	1	3	65.714	46	3	3	2
25.501	14	2	0	1	50.861	134	2	3	1	66.761	7	4	1	3
25.872	361	0	0	2	51.679	139	1	4	0	66.846	8	4	2	2
28.154	112	1	0	2	52.388	102	4	0	2	67.543	1	2	2	4
29.149	198	2	1	0	53.195	135	0	0	4	68.660	4	1	3	4
31.973	999*	2	1	1	54.479	4	1	0	4	69.176	1	1	0	5
32.275	437	1	1	2	56.218	54	3	2	2	69.593	2	6	0	0
33.157	542	3	0	0	56.790	2	5	0	0	70.188	16	1	5	2
34.167	245	2	0	2	56.988	6	1	1	4	70.694	2	4	3	0
35.697	31	3	0	1	57.364	25	3	1	3	71.204	2	5	0	3
38.473	2	2	2	0	58.215	1	2	0	4	71.378	2	1	1	5
39.364	51	2	1	2	58.490	12	5	0	1	71.963	5	4	0	4
40.110	203	1	3	0	58.678	8	4	1	2	72.209	29	4	3	1
40.726	30	2	2	1	59.234	3	3	3	0	72.469	15	2	0	5
40.852	19	1	0	3	60.434	43	2	4	0	72.877	27	5	2	0
42.290	41	1	3	1	60.891	30	3	3	1	73.380	3	3	3	3
42.528	29	3	0	2	61.806	33	1	2	4	74.373	17	5	2	1
43.928	52	1	1	3	62.071	12	2	4	1	74.459	23	2	4	3
44.722	13	4	0	0	63.418	83	5	0	2	75.205	7	2	3	4



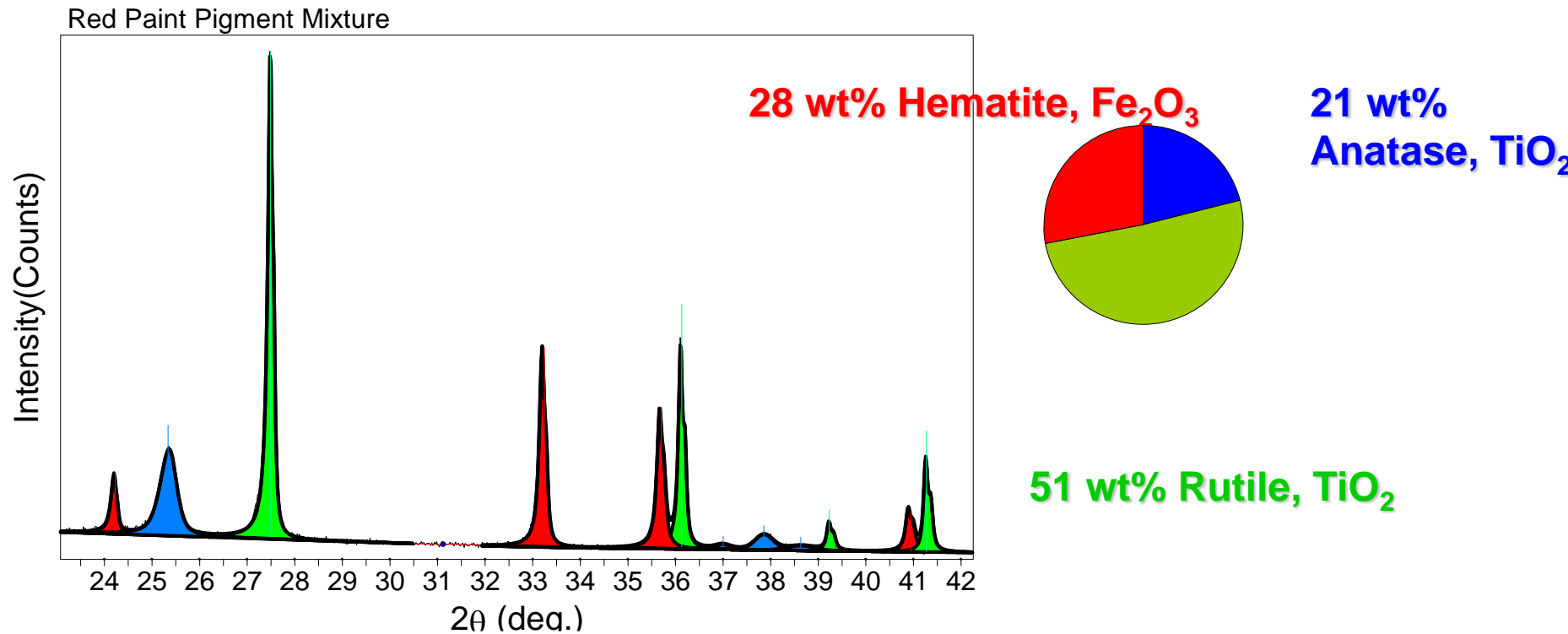
Quantitative Phase Identification

- In depth analysis for precise monitoring of phase composition

When

You know which phases are present and want to know how much

Example: You mix certain powders to make a red paint pigment then wonder how much of each is present



Relative Intensity Approach

- The ratio of peak intensities varies linearly with the ratio of weight fractions

$$\frac{I_{\alpha}}{I_{\beta}} = K \frac{X_{\alpha}}{X_{\beta}}$$

- K is determined from the mass absorption coefficients of all phases in the sample
 - Not the most accurate and reliable way
- Simplification of the ratio of the general intensity equation

Diagram illustrating the components of the general intensity equation $I_{(hkl)\alpha}$:

$$I_{(hkl)\alpha} = \left[\frac{I_0 \lambda^3}{64 \pi r} \left(\frac{e^2}{m_e c^2} \right)^2 \right] \left[\frac{M_{hkl}}{V_{\alpha}} \right] [F_{(hkl)\alpha}]^2 \left(\frac{1 + \cos 2\theta^2 \cos 2\theta_m^2}{\sin \theta^2 \cos \theta} \right) \left[\frac{X_{\alpha}}{\rho_{\alpha}(\mu/\rho)_s} \right]$$

Annotations:

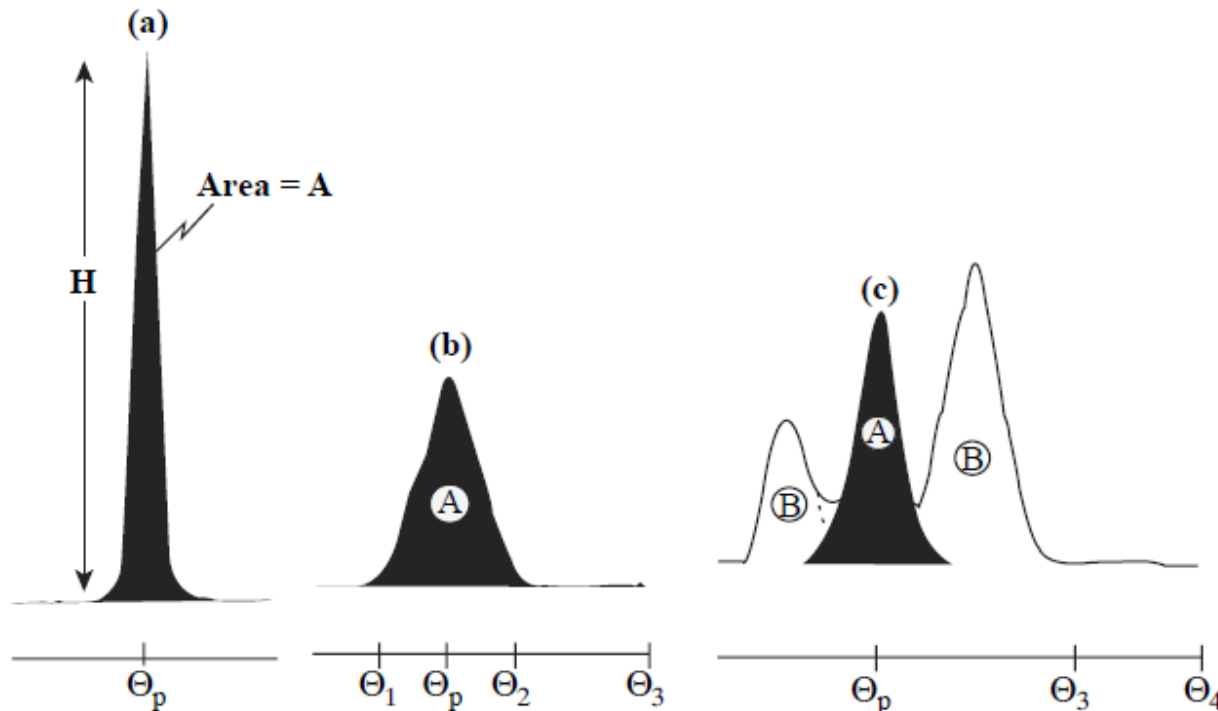
- Constant for an experimental setup: $\left[\frac{I_0 \lambda^3}{64 \pi r} \left(\frac{e^2}{m_e c^2} \right)^2 \right]$
- Multiplicity factor: $\left[\frac{M_{hkl}}{V_{\alpha}} \right]$
- Structure factor: $[F_{(hkl)\alpha}]^2$
- Lorentz polarization factor: $\left(\frac{1 + \cos 2\theta^2 \cos 2\theta_m^2}{\sin \theta^2 \cos \theta} \right)$
- Absorption factor: $\left[\frac{X_{\alpha}}{\rho_{\alpha}(\mu/\rho)_s} \right]$

$$I_{(hkl)\alpha} = \frac{K_e K_{(hkl)\alpha} X_{\alpha}}{\rho_{\alpha}(\mu/\rho)_s}$$

Relative Intensity Approach

- K can be determined :
 - by using published relative intensity values (I/I_c) → External Standard Method
 - empirically, by building calibration curves → Internal Standard Method
 - by simulating the diffraction pattern: whole pattern refinement → Reitveld Analysis
- To equate the concentration of a given phase with the intensity of the peaks in the multiphase pattern,
 - a single peak, → least precision
 - a number of peaks, → considerable precision
 - all of the peaks → ultimate precision (Reitveld analysis)can be utilized
- The shape of the peaks is decisive in choosing the intensity measurement method
 - When the diffraction peak is sharp and non-overlapped, peak height is taken as the representative value as it is considered proportional to the peak area
- Background corrected integrated intensities are measured by subtracting the background intensity from the peak height

- a) Peak height is proportional to peak area. Peak height is measured
 - b) Peak height not proportional to peak area. Combined intensity is calculated
 - c) Peak area is overlapped by other peaks
- This is a common case. Peak profiles should be adjusted



External Standard Method

$$W_{\alpha} = \frac{I_{\alpha}}{I_{\alpha}^P} \cdot \frac{(\mu/\rho)_m}{(\mu/\rho)_{\alpha}}$$

- The complexity of the analysis of multiple phases in a mixture is greatly reduced by referring all of the pure phase peak intensities to a single standard

$$\frac{W_{\alpha}}{W_s} = 1, \text{ so } \frac{I_{\alpha}^P}{I_s^P} = RIR_{\alpha} = \frac{(\mu/\rho)_{\alpha}}{(\mu/\rho)_s}$$

where I_{α} is the intensity of the 100% peak of phase α ,
and I_s is the intensity of the 100% peak of a reference
phase s , taken by convention to be α -Al₂O₃, corundum,
in a 50:50 mixture by weight

$$W_{\alpha} = \frac{I_{\alpha}}{I_s^P} \cdot \frac{(\mu/\rho)_m}{(\mu/\rho)_s} \cdot \frac{1}{RIR_{\alpha}}$$

$$W_{\alpha} = \left[\frac{I_{\alpha}^{hkl}}{I_{\alpha}^{REL}} \right] \cdot \left[\frac{(\mu/\rho)_m}{(\mu/\rho)_s} \right] \cdot \left[\frac{1}{I_s^P \cdot RIR_{\alpha}} \right]$$

Generalized External Standard Equation

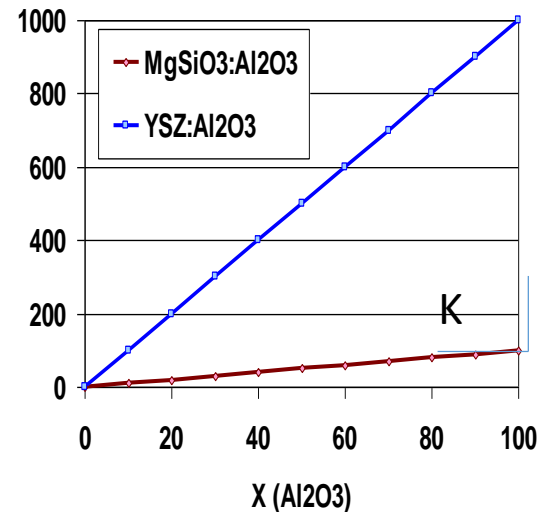
0-0.999

The use of RIR values is much faster and less prone to error than the determination of all of the weight fractions from the starting equation which would require reference to the integrated intensities of the 100% peak of each phase in its pure form

Internal Standard Method

- The proportionality constant K is determined empirically by building calibration curves from mixtures containing known quantities of internal standard

$$\frac{I_{\alpha}}{I_{\beta}} = K \frac{X_{\alpha}}{X_{\beta}}$$



Example – Three component mixture consisting of predetermined concentrations of MgSiO₃, YSZ, Al₃O₂

Sample	MgSiO ₃ wt%	YSZ wt%	Al ₃ O ₂ wt%
1	30	50	20
2	30	30	40
3	30	10	60
4	10	10	80

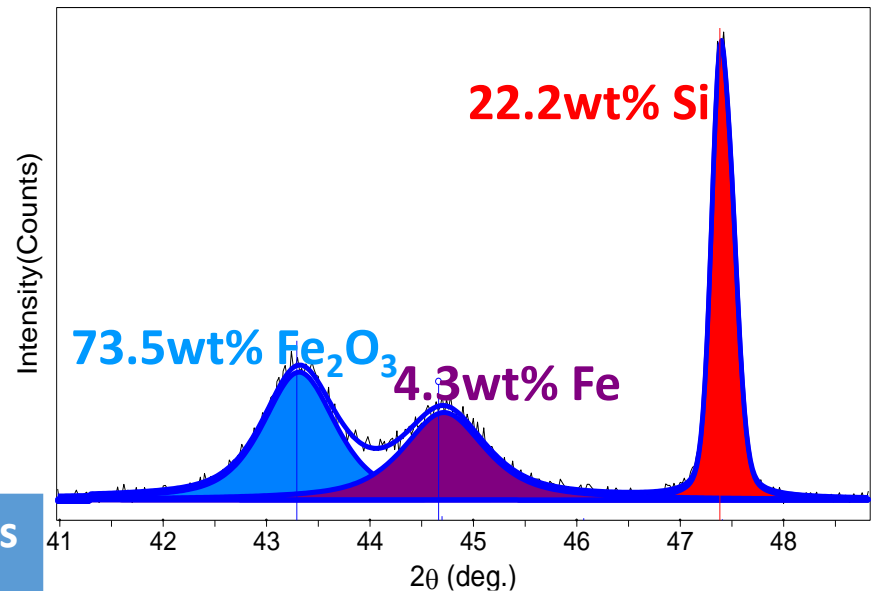
Determining Crystallinity using Standard Additions

- Adding a predetermined amount of well crystallized standard to a sample containing amorphous phases
- Calculate the observed amount of all phases using any of the quantitative analysis methods, ignoring the amorphous content
- Apply a normalization to all calculated amounts based on the initial standard amount

Example – Fe, Fe₂O₃ mixture with 17% Si standard

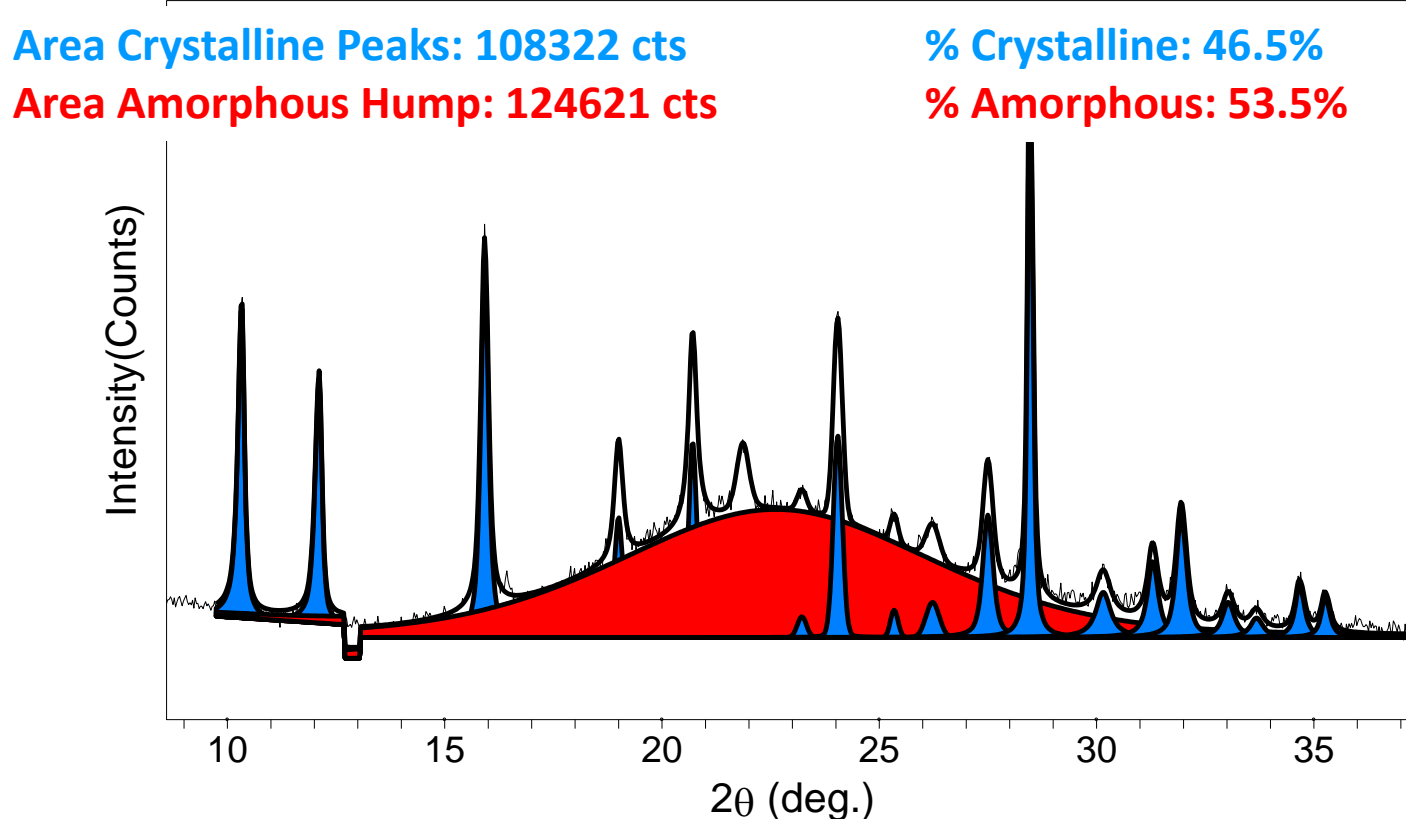
the discrepancy due to the amorphous content

Sample	Fe wt%	Fe ₂ O ₃ wt%	Si wt%	Amorphous
Calculated	4.3	73.5	22.2	0
Actual	3	55	17	25



Determining Crystallinity without Standard Additions

- If the mass absorption coefficient of an amorphous phase is the same as the crystalline content, the ratio of intensities can be used to determine % crystallinity

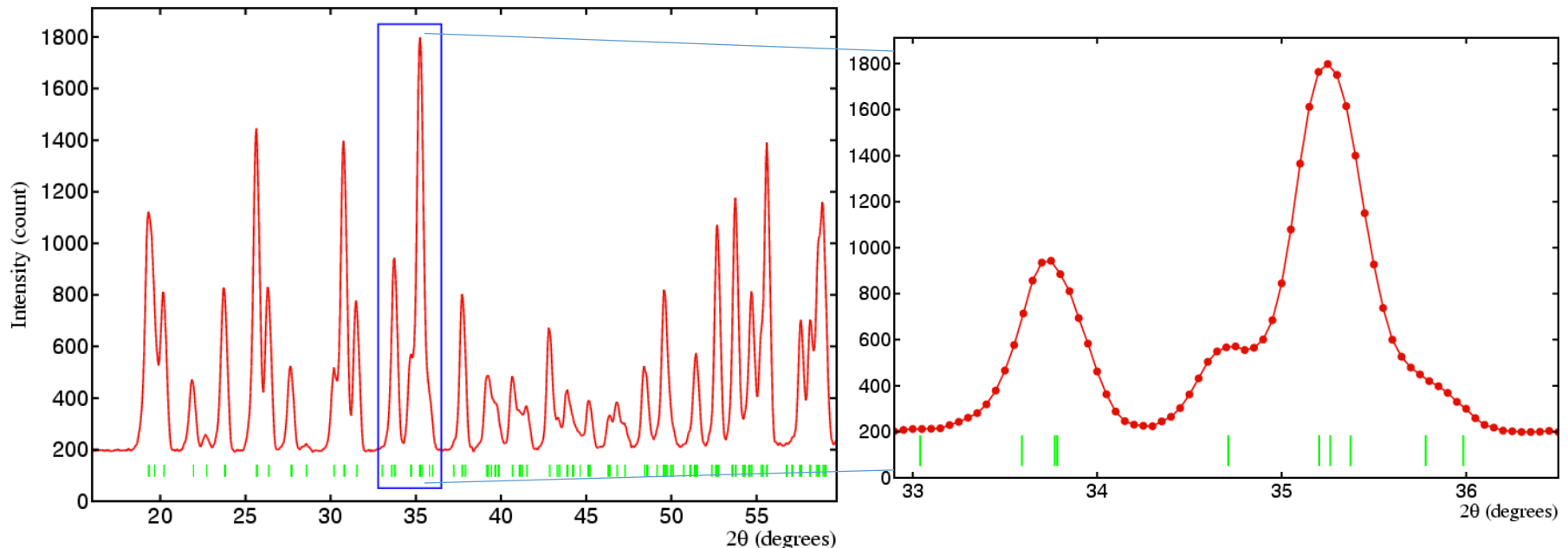


Rietveld Refinement

- Rietveld realised that the detailed profile of a powder-diffraction pattern contained a lot more information than the extracted intensities of composite peaks and stated that

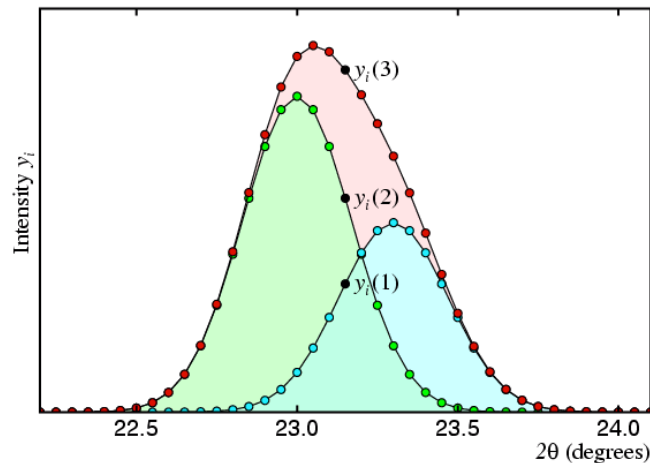
"The method of using the total integrated intensities of the separate groups of overlapping peaks in the least-squares refinement of structures, leads to the loss of all the information contained in the often detailed profile of these composite peaks. By the use of these profile intensities instead of the integrated quantities in the refinement procedure, however, this difficulty is overcome and it allows the extraction of the maximum amount of information contained in the powder diagram."

H. M. Rietveld



Reitveld Refinement

- Reitveld found that the detailed profile could be fitted on a point by point basis using the simple Gaussian peak-shape function without any need to extract intensities of composite groups of reflections

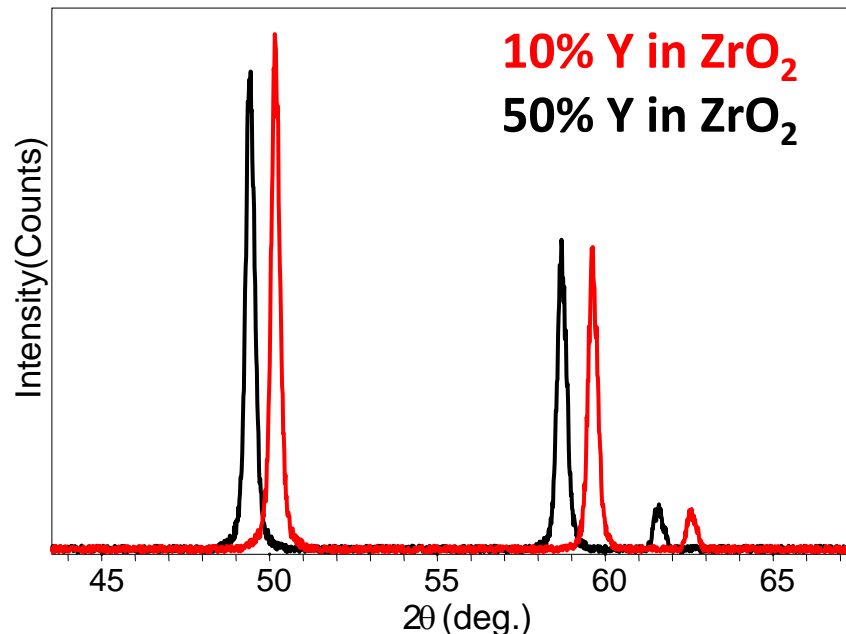


- Refinement implies taking an approximate model of the structure and converting it so that diffraction data calculated from the model structure has a closer resemblance to the measured data
- Many complex computer algorithms operating on the basis of using a least-squares procedure to refine the initial structure model in order to improve the agreement between the observed diffraction data and that calculated from the model
- In other words Reitveld refinement is a broad numerical technique used to make the XRD pattern as true and error-free as possible

Lattice Parameter Determination

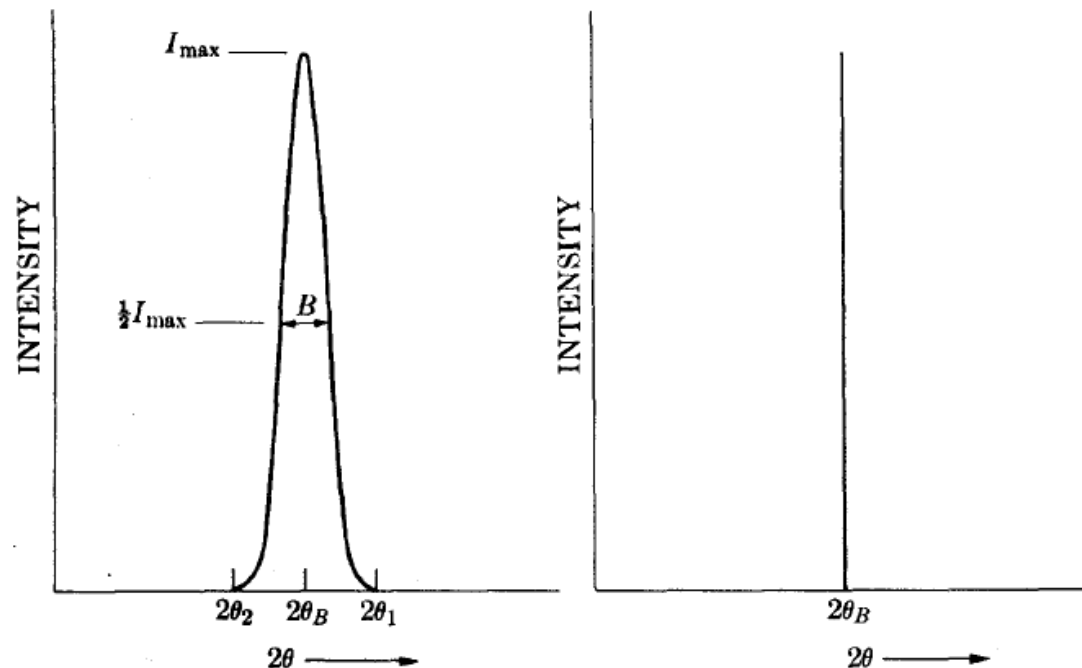
- Changes in the interplanar spacing of a material such as substitution of an atom can be detected by XRD
- The position of the diffraction peaks are a product of the space between planes of atoms
- The change in peak intensity due to substitution of atoms with similar Z is subtle, however peak angles which indicate the interplanar space change

Example – Substitution of Zr in YSZ with Y



Crystallite Size Broadening

Ideally a peak in the Bragg diffraction pattern is a line without a width. In reality the peaks have some width that originate from the instrument and the size of the crystals



Crystallite Size Broadening

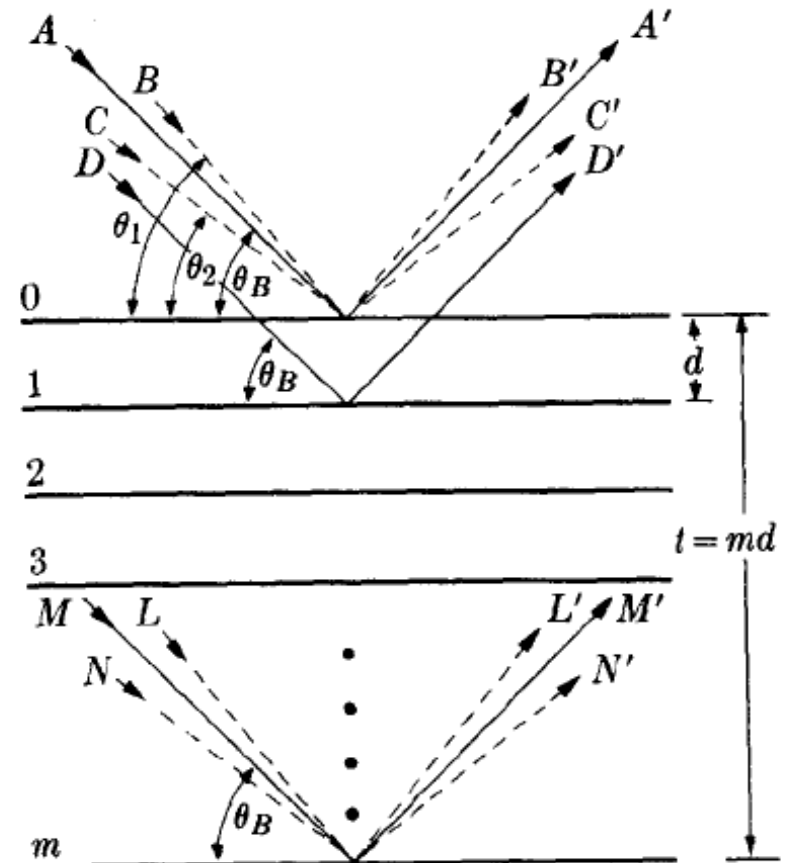
A diffraction peak is produced by a plane with θ_B that satisfies Bragg's law

However X-ray beam in para-focusing geometry is not perfectly parallel and is diffracted at angles between θ_1 and θ_2 . So X-rays scattered at angles between θ_1 and θ_2 do not interfere with each other perfectly constructively or destructively

In this case X-rays scattered from plane 0 and 1 have angles θ_1 that correspond to Bragg's number n less than an integer

These X-rays will interfere with X-rays scattered from plane m that scatters rays with an angle that corresponds to $n/2$ (out of phase - destructive interference)

For very fine crystals the plane m is not present so that X-rays diffracted at θ_1 will be detected as peak broadening

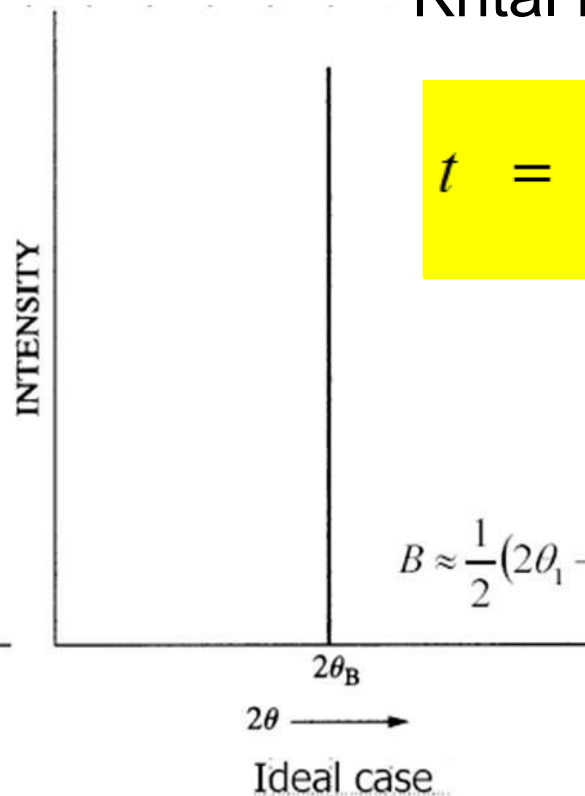
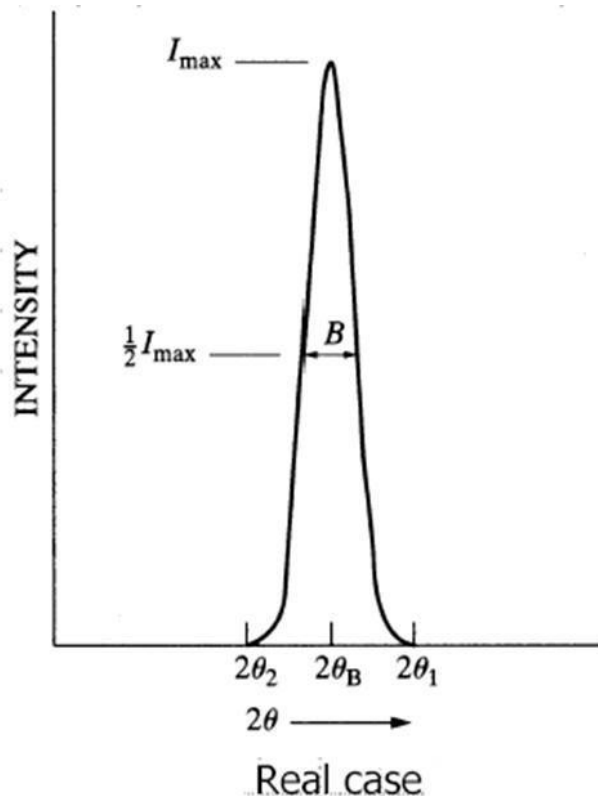


Kristal Boyutu

Peak genişliği ile kristal boyutu ters orantılıdır. Peak genişliği (B) arttıkça kristal boyutu küçülür.

Krital Boyut hesabı (t) :

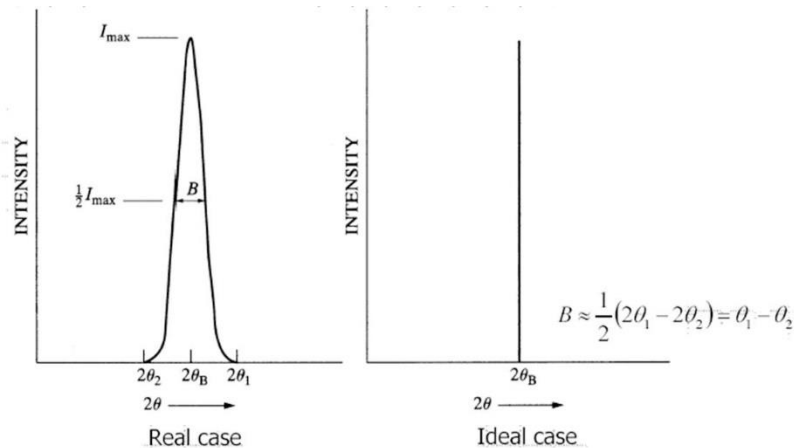
$$t = \frac{0.9 \lambda}{B \cos \theta_B}$$



$$B \approx \frac{1}{2}(2\theta_1 - 2\theta_2) = \theta_1 - \theta_2$$

Crystallite Size Broadening

- No crystal is perfect due to its finite size
- The deviation from perfect crystallinity leads to a broadening of the diffraction peaks which is negligible above a certain size ($\sim 0.1 - 1$ micron)
- Peak broadening occurs for powder samples with crystallites around 100 nm in diameter



$$L = \frac{K\lambda}{B(2\theta)\cos\theta} \quad K=0.87-1.00$$

Sherrer Equation for samples with fine crystallites

- Peak width is measured at the half of the maximum peak intensity (FWHM)
- Various methods to determine the crystallite size of an unknown fine powder mixed with a coarse standard powder

$$B^2 = B_U^2 - B_S^2$$

Warren's equation – the simplest form

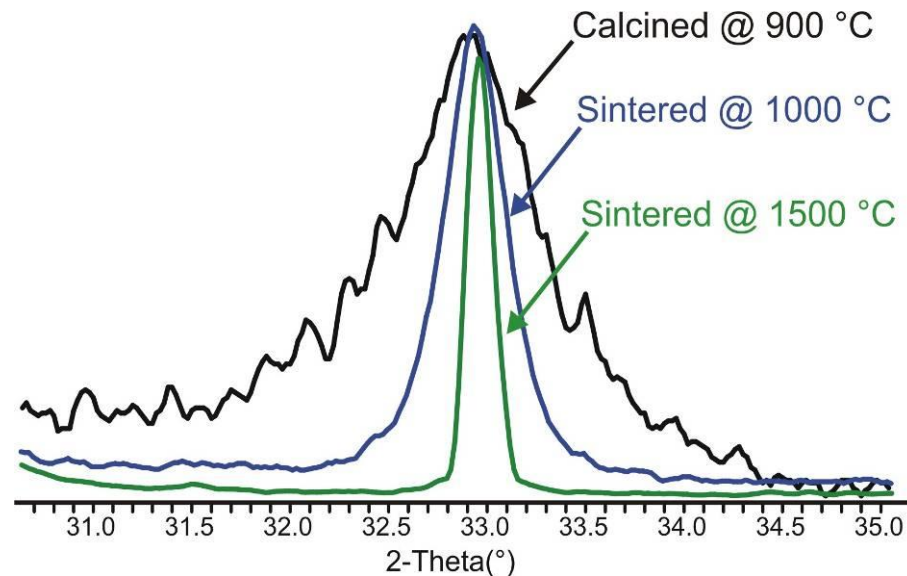
Crystallite Size Broadening

Example - Sherrer analysis of the main peak for a heat treated polycrystalline $\text{La}_2\text{Zr}_2\text{O}_7$ to estimate the crystallite size

Calcined at 900 °C for 1 hr: ~10 nm average crystallite size

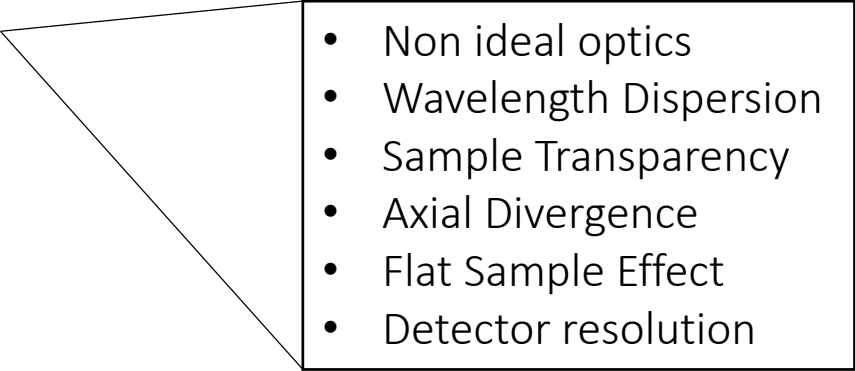
Sintered at 1000 °C for 96 hrs: ~30-40 nm average size

Sintered at 1500 °C for 2 hrs: >100 nm average crystallite size



Factors That Contribute to Crystallite Size Broadening

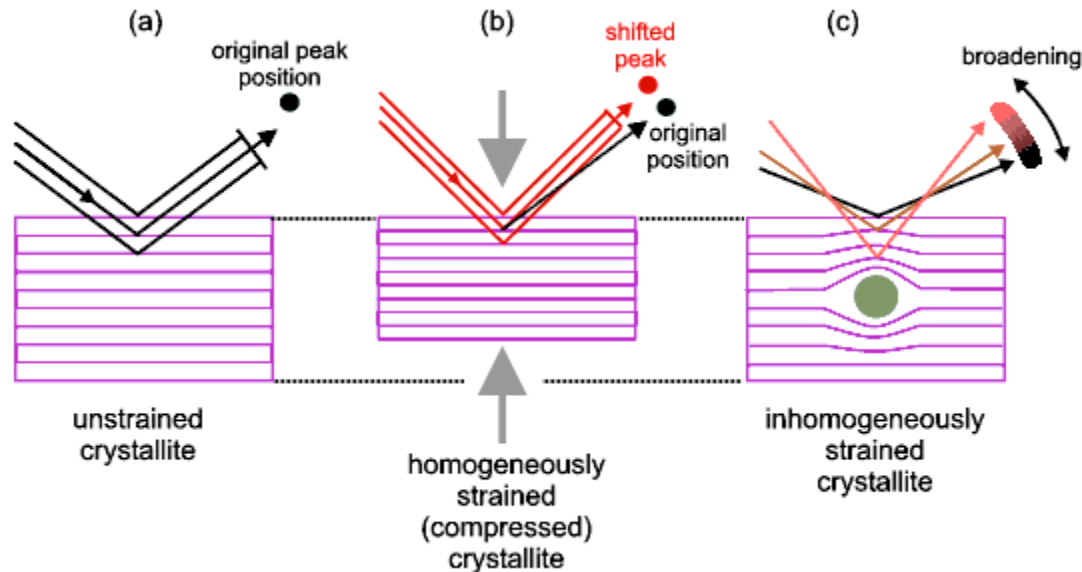
- Instrumental Broadening
- Crystallite Size
- Microstrain
- Faulting
- Dislocations
- Antiphase Domain Boundaries
- Grain Surface Relaxation
- Solid Solution Inhomogeneity
- Temperature Factors

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- Non ideal optics
 - Wavelength Dispersion
 - Sample Transparency
 - Axial Divergence
 - Flat Sample Effect
 - Detector resolution

- The peak profile is a combination of the profiles from all of these contributions
- Peak broadening analysis is most accurate when the broadening due to crystallite size effects is at least twice the contribution due to instrumental broadening

Microstrain Determination

- Inhomogeneous strain is the second main source of specimen broadening

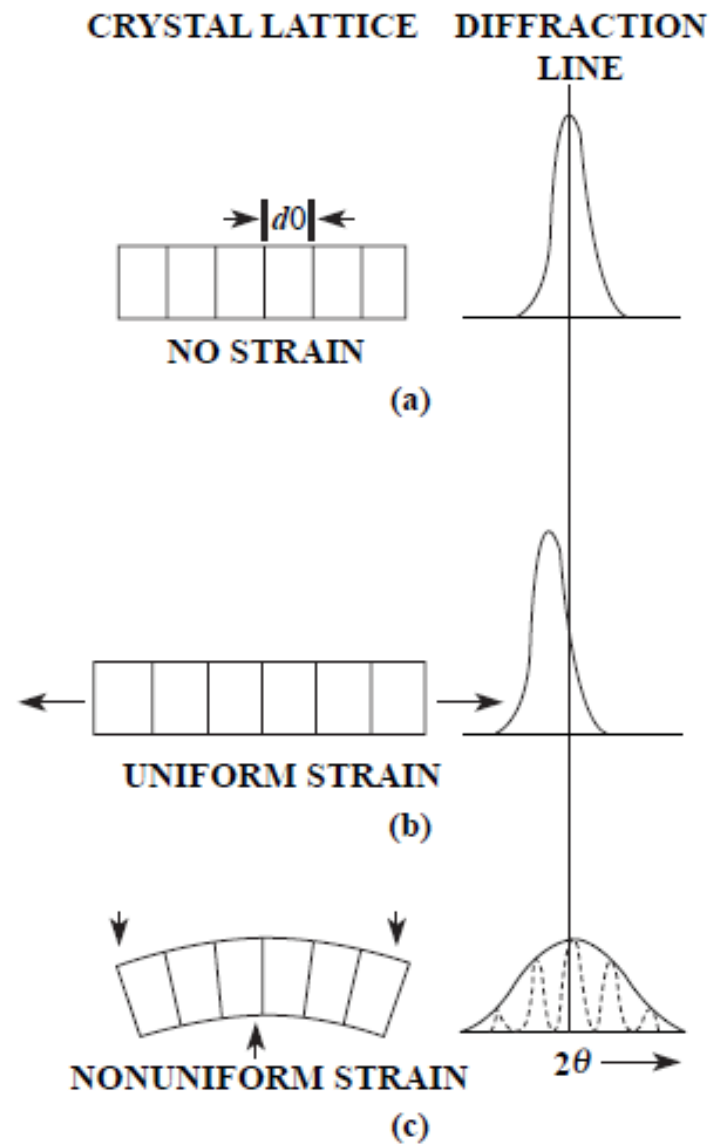


a compressive stress would make the d spacings smaller and a tensile stress would make the d spacings larger which would cause only peak shifting

if the strain is *inhomogeneous* then different crystallites will be strained by different amounts and the shifts in 2θ will be variable resulting in peak broadening

$$B(s) = C \tan\theta, \quad C=4-5$$

Microstrains



Seperating the size and microstrain contributions to peak broadening

- Williamson and Hall simplified the contributions of size and strain to a basic sum or sum of squares

$$B(2\theta) = \frac{K\lambda}{L \cos \theta}$$

$$B(s) = C \tan \theta$$

$$B(\text{total}) = B(2\theta) + B(s) = \frac{K\lambda}{L \cos \theta} + C \tan \theta$$

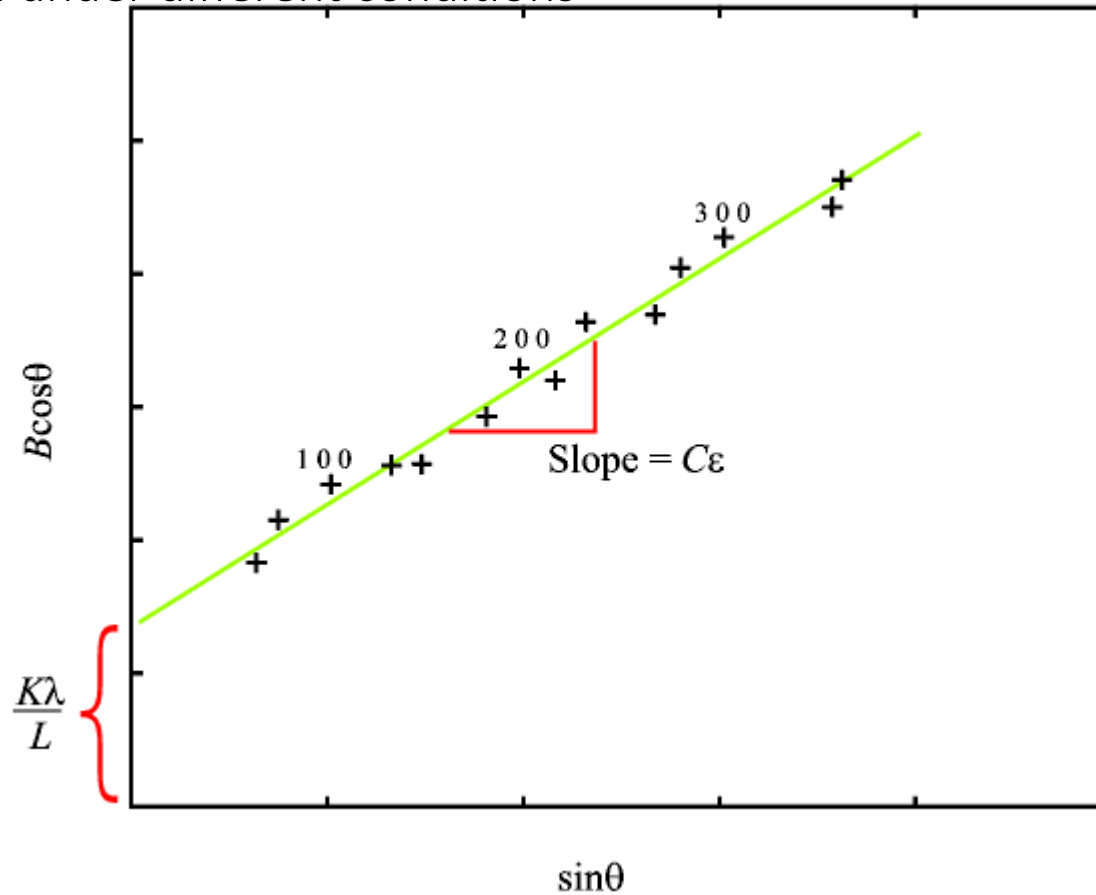
$$B(\text{total}) \cos \theta = \frac{K\lambda}{L} + C \sin \theta$$

By plotting a Williamson-Hall graph, where $B(\text{total}) \cos \theta$ and $\sin \theta$ are the axes, the strain component is obtained from the slope (**C**) and the size component from (**L**) the intercept

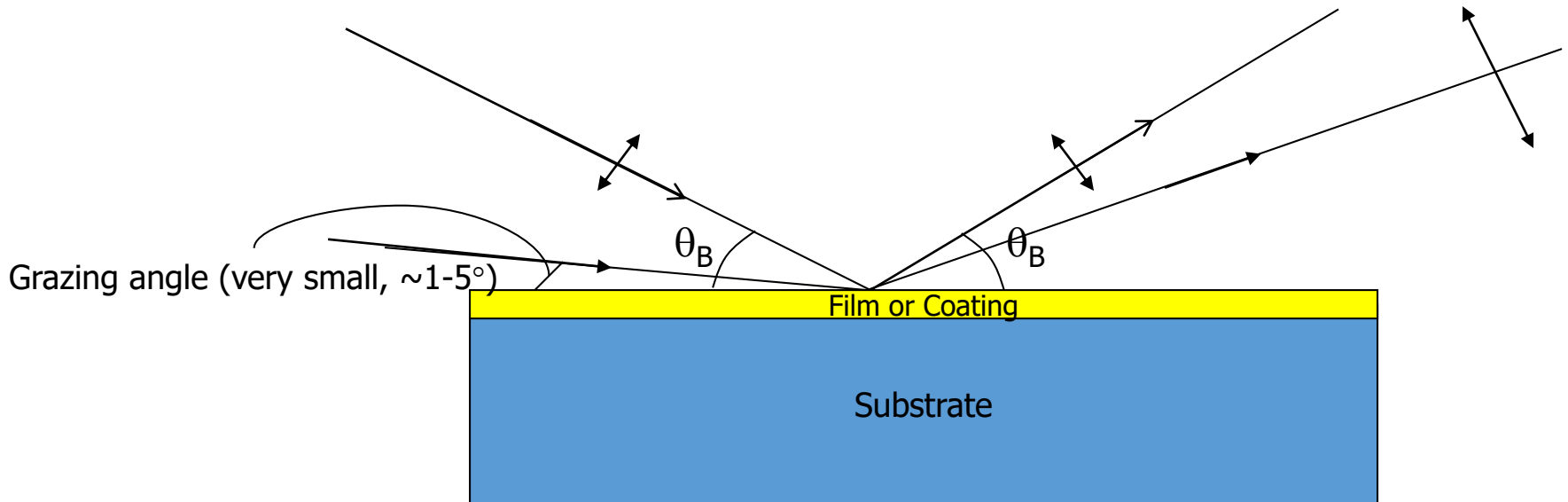
Separating the size and microstrain contributions to peak broadening

- Williamson-Hall method lacks precision due to its simplifications but it can be a useful method if used to observe trends in the relative sense.

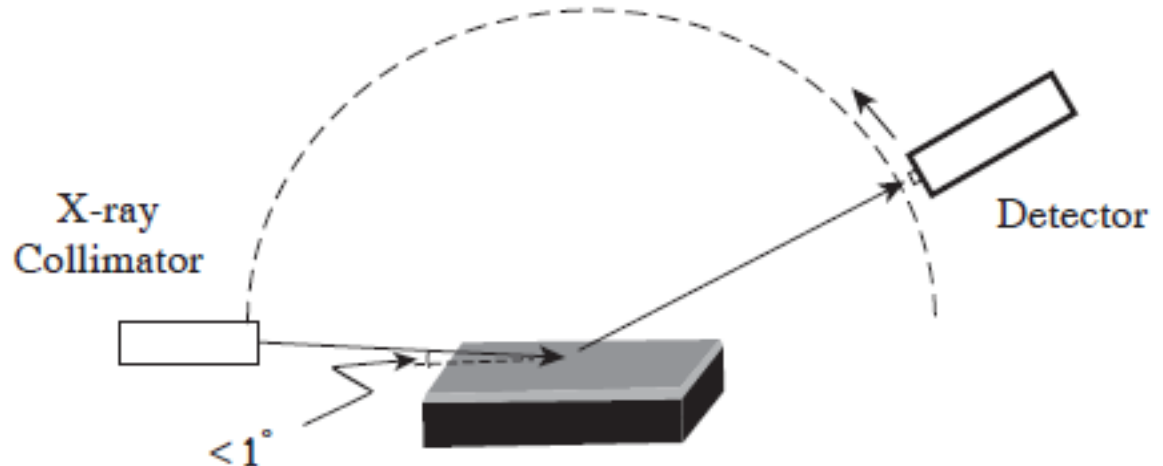
Example - a study of many powder patterns of the same chemical compound, but synthesized under different conditions



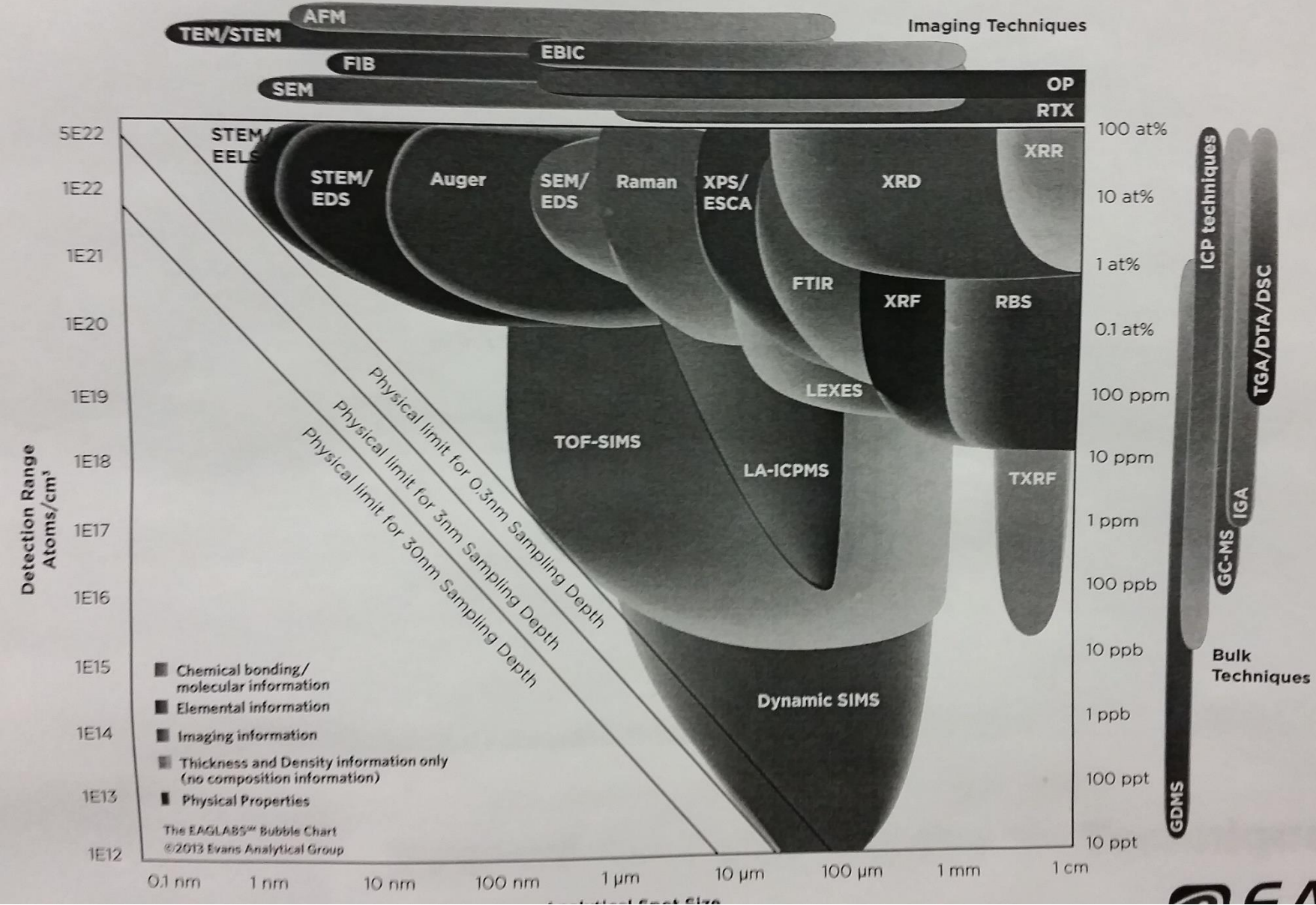
Thin Film Specimen

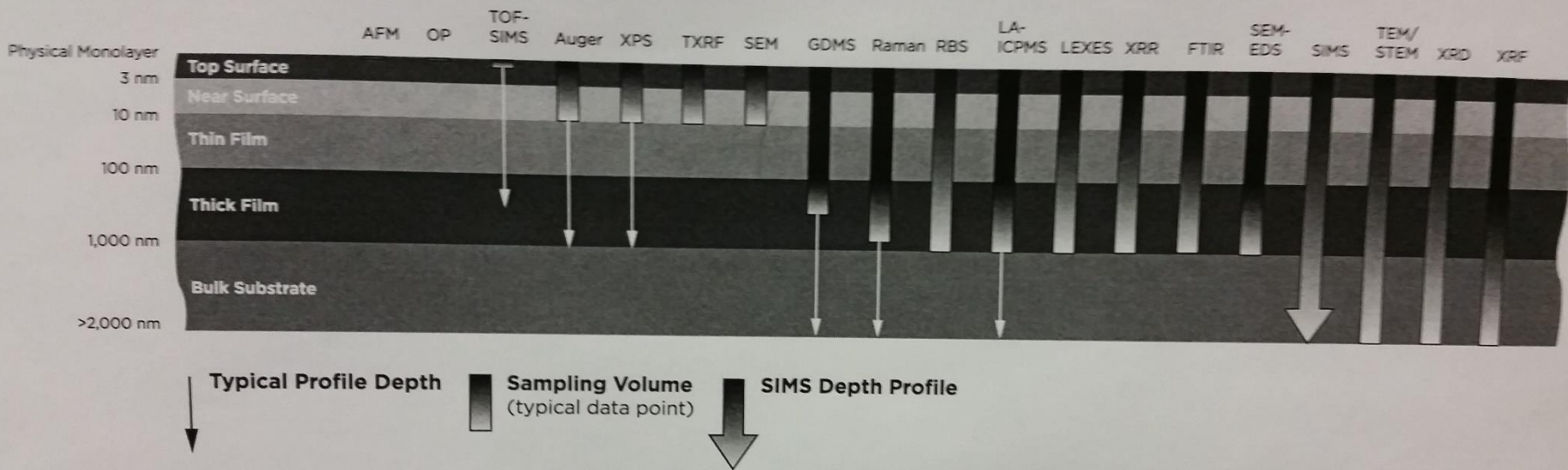


- Smaller volume *i.e.* less intensity of the scattered beam from the film
- Grazing angle
 - Useful only for polycrystalline specimens



Analytical Resolution VS Detection Limit





Data collection and analysis procedure

- Choose 2θ range
- Step size and time per step
- Hardware: slit size, filter, sample alignment
- Fast scan followed with a slower scan
- Look for fluorescence
- Collected data: Background subtraction, $K_{\alpha 2}$ stripping
- Normalize data for comparison I/I_{\max}

Thank you for your interest!

The following sources provide more information on X-Ray
Diffraction:

Elements of X-Ray Diffraction (The Bible)
by B. D. Cullity

Fundamentals of Powder Diffraction and Structural
Characterization of Materials
by V. K. Pecharsky et al.

Principles and Applications of Powder Diffraction
by A. Clearfield et al.